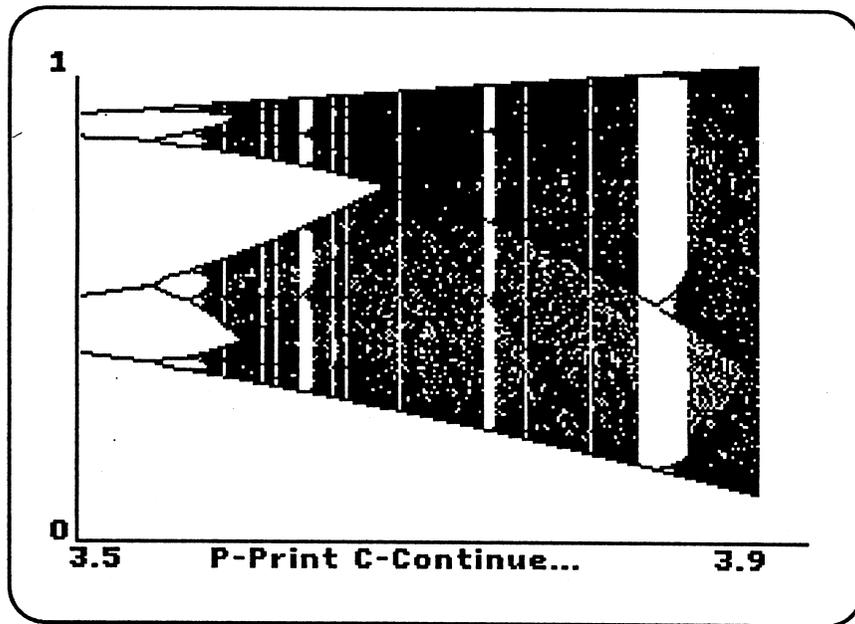

CHAOS Plus...

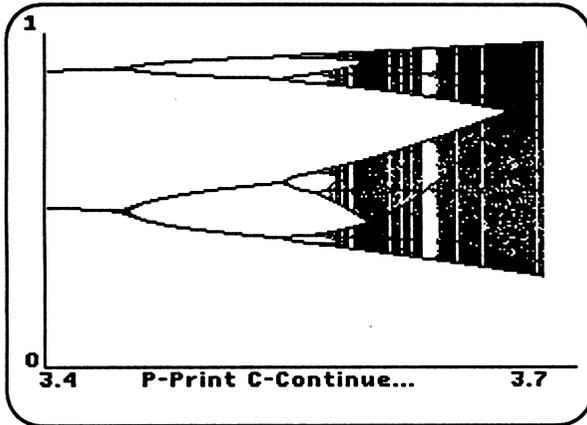
Instructions & Activities



EduTech 1927 Culver Road, Rochester, NY 14609
Innovative Educational Software

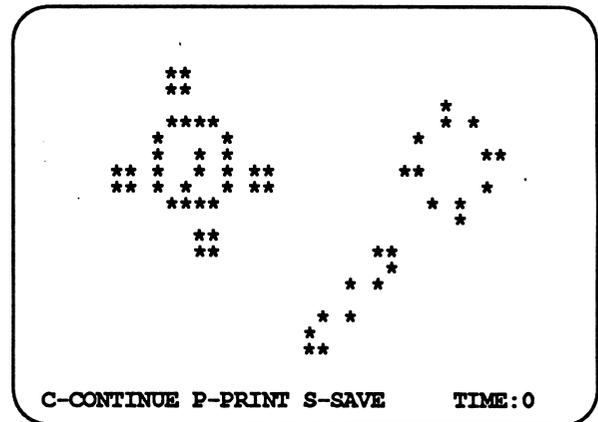
Overview

EduTech's **CHAOS Plus...** is an interactive introduction to chaos, fractals and cellular automata. Many magazines and television shows have presented these ideas to the public. Biology, chemistry, physics, math, economics - the areas of study affected by chaos and fractals are without limit. The eye-catching beauty of the graphic output stirs the imagination. This collection of programs encourages students of all ages to explore these concepts.



Chaos occurs when small changes in initial conditions or equation parameters produce unpredictable (but repeatable) results.

The Game of Life is an example of two-dimensional cellular automata. A few simple rules controlling the birth, death and survival of cells in a checkerboard type environment determine the appearance of all future generations



Fractals are rough lines or figures which remain rough even under microscopic examination. The Mandelbrot Set uses a simple math formula to produce one of the most complex graphs in mathematics.

**Full directions for all activities
are included in this manual.**

This program could not have been created without a great deal of help. Orest Mankiwskyj supplied innumerable reference books and constant moral support. Without his help this program would not exist. Alan Cromer supplied initial technical advice. Joel and Anne Huberman introduced me to Gleick's book on Chaos, showed me some beautiful color fractals on their Mac II, and helped with the proof-reading. R. W. W. Taylor helped with programming tips on the Mandelbrot Set. Richard Elberfeld made numerous suggestions that improved the user interface. Susan Kurtz, Heide Elberfeld, Neil Reznikoff, and Joe Froehlich all made significant contributions. Also Phyllis Mandell and the Jericho High School staff.

Three Body Orbits, part of EduTech's AstroLab series, provided the backbone of the three body orbit section. The AstroLab series provides more flexibility and more detail for the study of planetary motion. The portions of the program included in Chaos are used with the permission of the author, Alan Cromer.

This manual was written by John Elberfeld. It was edited and type-set using PageMaker on a Macintosh.

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EduTech TECHNICAL NOTES

EduTech disks function on the Apple II-Plus, Apple //e, Apple //c, Apple //GS and most Apple II compatible computers. Because of the variety of Apple compatible computers, we can not guarantee that all programs will run on all makes and models. (If you have problems, please let us know immediately.)

Back-up disks are provided at the time of sale for all EduTech packages. Lab Packs and quantity discounts provide schools with an economical alternative to breaking the law. Contact your software dealer or EduTech for details.

WARRANTY

If, within two years of purchase, a disk is damaged for any reason, it will be replaced free of charge. For an immediate replacement, send the damaged disk and proof of purchase to:

EduTech
1927 Culver Road
Rochester, NY 14609
(716) 482-3151

Replacement disks are also available in emergency situations and after the two year warranty expires. Call for more details.

Game paddles or Interface Equipment are required for certain EduTech packages. Replacement parts are available from EduTech. Color Monitors and "Lower Case" capabilities enhance the operation of some of the programs.

NOTE: Interface Equipment and Game Paddles duplicate the function of the Open and Closed Apple Keys. **If your system fails to boot with the interface equipment attached, remove the photogates or remove the equipment completely.** Your computer may go into a self-diagnostic mode, thinking you are holding the Apple keys down.

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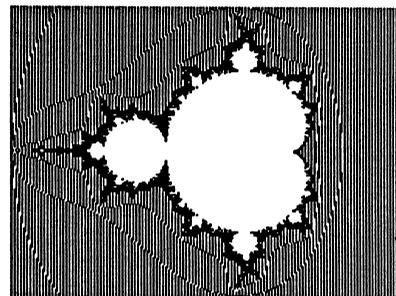
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Some programs were written using help from Beagle Bros. Inc., 3990 Old Town Ave., San Diego, CA 92110

Introduction to CHAOS Plus...

Chaos, fractals, cellular automata, bifurcation diagrams and strange attractors are science topics that were non-existent or barely discussed a few years ago. The beauty of the computer generated graphs created to display properties of these concepts has captured the attention of many students who might otherwise ignore scientific progress. EduTech's CHAOS Plus... helps both students and teachers become familiar with these new and exciting concepts.

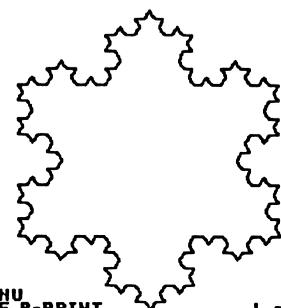


"In Newtonian mechanics, the motion of a mechanical system, such as that of the earth about the sun, is determined by mathematical equations. The solution of the equations is a relationship that describes where the objects of the system will be in the future, given their current positions. For this reason, the notion of determinism and predictability have been thought to be synonymous. With the advent of the computer, however it has been found that some systems, while changing in time in a totally determined manner, are unpredictable because they are infinitely sensitive to the conditions specified at the start. Such systems are said to be chaotic... This is happily one of those ideas that, while obscure to the mathematician, is clear to the layman... Minute changes in your life at an instant, can have profound consequences for your life's trajectory. This is what the mathematician means by chaos." (Cromer)

"There is order in chaos: randomness has an underlying geometric form. Chaos imposes fundamental limits on prediction, but it also suggests causal relationships where none were previously suspected" (Crutchfield)

For example: When examining the motion of Hyperion, a moon of Saturn, astronomers discovered irregular motion. They could NOT predict the motion of the moon, even though it obeyed Newton's Laws of Motion, and no outside influences were affecting its motion. When studying the motion of the moon, they uncovered "Something much more remarkable. An inherent feature of mathematical equations in dynamics. The ability of even simple equa-

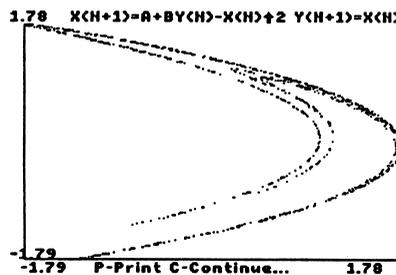
What is Chaos?



ESC-MENU
E-ERASE P-PRINT

L = 711

Learning More about Chaos



tions to generate motion so complex, so sensitive to measurement, that it appears random. Appropriately, it's called *chaos*." (Stewart)

This manual will not attempt to provide a complete introduction to chaos. The bibliography provides an extensive list of reference books for those desiring more information than can be provided here.

This set of programs helps students explore the concepts behind chaos. The emphasis is on trial and error, discovering, creating, testing... not facts, figures and definitions. Respected scientists have spent years studying the concepts presented here, even though junior high school students are capable of making their own meaningful discoveries in a class period.

The relationship among the concepts may seem vague at first. All the programs try to show that extremely simple rules that everyone can understand can produce complicated results that few if any people understand. Changing the initial or starting conditions in a situation by a small amount can produce unpredictable and overwhelming changes later on. On the other hand, some initial conditions have little or no effect on the general behavior of other sets of equations. EduTech's CHAOS Plus... encourages students to explore these ideas.

Getting Started

These programs are carefully constructed to introduce students to Chaos. The time required to work through each exercise depends on the curiosity and ability of the student. Some students may require encouragement to experiment and test their ideas. Disks are in the DOS 3.3 Apple format, and work on the entire family of Apple II computers. A single drive and monochrome monitor are the minimum required, but two disk drives and a color monitor will increase the appeal of the programs. Any method used to increase the processing speed of the Apple will also help some of the programs.

Every effort has been made to create a "bug-free" program. However, extensive vandal-proofing was omitted to allow students to explore the construction and programming techniques used within the programs.

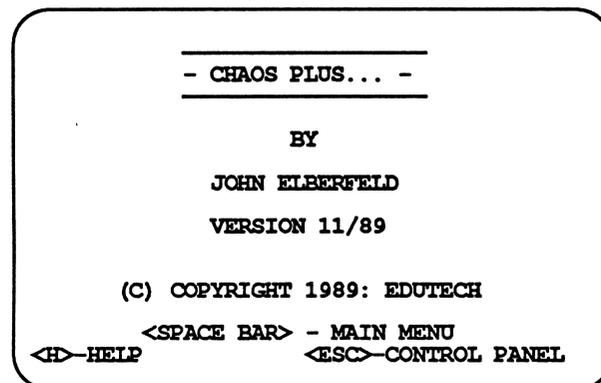
The programs were designed for use with an Apple II+, Apple IIe, IIc, IIGS or other compatible computers. The Program Disk is full and can not be used to store data. Do NOT *write protect* the data disk. A Data Disk is provided with sample graphs and data stored on it.

Insert the Program disk in Disk Drive 1, your "boot-up" drive. In Apple IIC's, insert the disk in the internal drive. Turn on the monitor and then the computer. The computer will automatically display the EduTech Logo and then the title page. Press the Escape Key <ESC> to set up the program to match the computer hardware that it will be run on.

Hardware Required

Starting Up

Title Page



Program Control Panel

```
*****
*
*          PROGRAM CONTROL PANEL
*
* 1) DATA DISK DRIVE : 2
*
* 2) GRAPHICS COMMANDS : II GS
*
* 3) INITIALIZE DATA DISK
*
* 4) LOWER CASE DISPLAY : Available
*
* 5) DISPLAY MAIN MENU
*
*****
NUMBER OF ITEM TO CHANGE :
```

At this time the location of the data disk is indicated, the type of graphics printer is selected, data disks are initialized, and the type of text display available is indicated. Sending data to non-existent drives or printers can cause unpredictable results, so this information must be accurate.

Data Disks

Data disks are standard Apple DOS 3.3 disks with several key text files initiated to "0". You MUST initialize data disks using this option or else the program will produce endless error messages. You are encouraged to have several blank data disks on hand to store data. The graphs tend to fill up the disks quickly, and the current graphs are lost if the program is rebooted to initialize a data disk.

Backups of the Program Disk and the Data Disk are provided in the original package. Permission is granted to create additional backup copies of the Program Disk to be used ONLY if the original Program Disks are damaged. Students may make temporary copies for programming exercises. Lab Packs and licensing agreements are available if multiple copies of the Program Disk are needed for classroom use. These options are inexpensive and *legal*.

Graphics Printer Selection

```
*****
*
*          INTERFACE SELECTION
*
* 1) GRAPPLER+
* 2) SILENTYPE
* 3) PKASO
* 4) IMAGEWRITER/SUPER SERIAL
* 5) IMAGEWRITER/II C
* 6) IMAGEWRITER/II GS
* 7) CREATE PRINT COMMANDS
* 8) NO MATCH
* 9) RETURN TO CONTROL PANEL
*
*****
1, 2, 3, 4, 5, 6, 7, 8, OR 9?
```

Printing Graphics

Many of the graphs are attractive and some involve hours of calculations. Printing out the graphs on paper can provide students with information to put in reports. Not all printers can handle graphics, and not all printers that can do graphics can be accessed by this program. If your specific printer is not listed, try some of the options. Many cards are "Grappler Compatible," and some printers have "ImageWriter Simulation" modes.

Information presented on the Program Control Panel is stored on the Program Disk. The disk **MUST NOT** be write protected.

The Program Control Panel is selected only if changes are made to the computer hardware. In general, students will simply press the <SPACE BAR> to continue.

After the title page, the Disk Contents menu appears. Here the students choose the topic they wish to investigate.

```
*****
*                                     *
*           DISK CONTENTS             *
*                                     *
*   1)  CELLULAR AUTOMATA            *
*   2)  THE GAME OF LIFE              *
*   3)  GRAPHS OF CHAOS              *
*   4)  ATTRACTORS                   *
*   5)  FRACTAL LINES                *
*   6)  RANDOM WALKS                 *
*   7)  THE MANDELBROT SET           *
*   8)  THREE BODY ORBITS           *
*   9)  END                           *
*                                     *
*****
1, 2, 3, 4, 5, 6, 7, 8, OR 9?
```

Disk Contents

The rest of the manual will explain each of these selections in detail.

In most programs, press the <SPACE BAR> to continue on in the program. Press the Escape <ESC> key to stop or back up. Because it takes time to see if a key has been pressed, some sections do not respond instantly to the escape key, or wait until a section is completed before letting you return to the menu. Most other prompts are listed across the bottom of the screen.

Entering Data

Entering data is fast and easy. Just type in a number within the given range. If a number is out of that range, the computer will not let you press the <RETURN> key- it will just "beep" at you. You must press the back arrow key and type in an acceptable number. If an acceptable number appears on the screen (usually the previous value selected for that parameter), all you need to do is hit the <RETURN> key.

Program Bombs

Because so much math is involved, it is possible to run into some problems. Sometimes a certain combination of parameters will produce a Division by Zero error. If this happens, simply type RUN or reboot the disk. Checking for potential errors slows down the programs, so not all possible errors are monitored all the time. Program "bombs" may happen. This is a trade off decision.

Cellular Automata

"Cellular automata are not real things, they're only abstractions, creatures of the intellect. But they're big with Wolfram and his cohorts because it turns out that, when these imaginary mechanisms are simulated by a computer, they replicate the operations of physical systems actually found in nature... Later he discovered that cellular automata not only produced the patterns found on seashells, they also simulated the structure of snowflakes, the growth of crystals, the meandering of rivers, and any one of a dozen things. It was incredible. In went a few lines of computer code, out came the real world, as if by magic. Cellular automata, Wolfram decided, might be able to explain the very architecture of nature." (Regis)

Introduction

```
*****  
*  
*      LINEAR CELLULAR AUTOMOTA      *  
*  
*      1)  CREATE STARTING PATTERN    *  
*      2)  RECALL RESULTS              *  
*      3)  DESCRIPTION                 *  
*      4)  RETURN TO DISK CONTENTS    *  
*  
*****  
1, 2, 3, OR 4
```

Main Menu

One Dimensional Cellular Automata are really a row of cells or boxes. You may assign each box a number between 0 and 3 as an initial condition at time 0. Each time period, every box receives a number based on the sum of its value and the values of its two bordering neighbors. Rules are written to determine the number assigned the box or cell based on the sum.

```
CELLUALR AUTOMATON-LINEAR FORMAT  
-----1-----  
  
ENTER YOUR STARTING VALUES BY TYPING  
THE NUMBERS 1, 2 OR 3 IN THE SPACES  
IN THE LINE ABOVE. A SPACE WITH NO  
NUMBER RECEIVES A '0' VALUE.  
  
USE ARROW KEYS TO MOVE DOWN THE LINE.  
PRESS <RETURN> WHEN DONE.  
  
ENTER 'R' TO PRODUCE A RANDOM LINE.
```

Starting Values

Rule Creation

CELLULAR AUTOMATON-LINEAR FORMAT

-----1-----

0 1 2 3 4 5 6 7 8 9 = SUM
 - 3 1 2 - - - - - = NEW CELL VALUE

THE NEW VALUE OF A CELL DEPENDS ON THE
 SUM OF THE VALUES OF THE CELL AND ITS
 TWO IMMEDIATE NEIGHBORS.

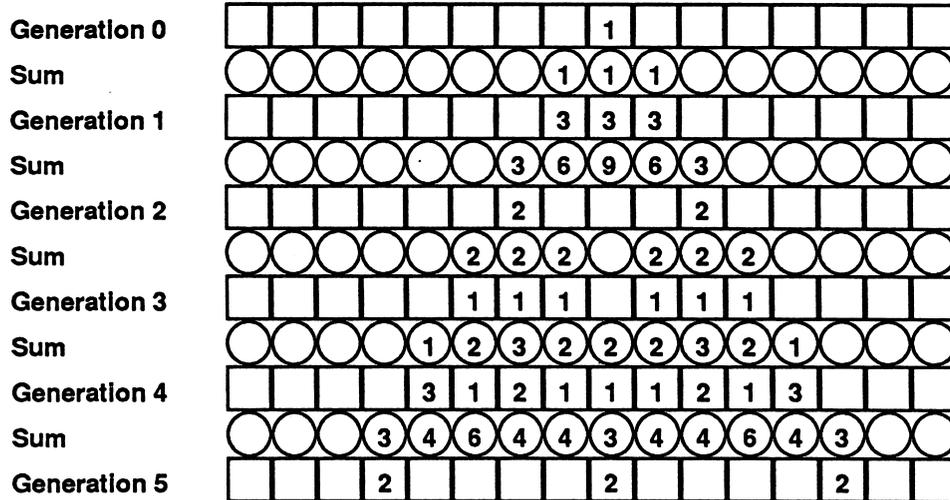
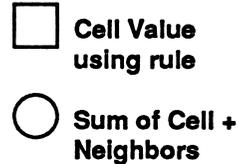
BELOW EACH SUM ENTER 0,1,2 OR 3 AS THE
 NEW CELL VALUE BASED ON THAT SUM.

The same procedure is followed for entering both the initial values and the rule. Pressing the arrow keys will move the cursor back and forth across the data line. When the cursor is in the proper location, press 0, 1, 2 or 3. The computer will generate a random initial condition or a random rule if the letter "R" is pressed.

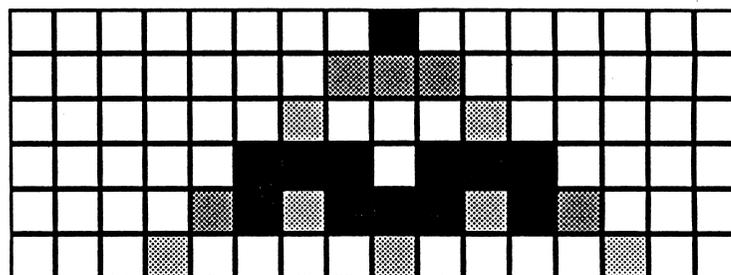
An Example

Rule

Sum 0 1 2 3 4 5 6 7 8 9
 New Value 0 3 1 2 0 0 0 0 0 0



How the Screen Looks



One of the leaders in this field has a concise description of this process:

What Does the Program Do?

"Cellular automata are simple models that appear to capture the essential features of a wide variety of natural systems. A one-dimensional cellular automaton is made up of a line of cells, shown (on the screen) as colored squares. Each cell can take on a number of possible values, represented by different colors. The cellular automaton evolves in a series of steps, shown as a sequence of rows of squares progressing down the (screen). At each step the values of all the cells are updated according to a fixed rule. The rule specifies the new value of a cell in terms of the sum of its previous value and the previous values of its immediate neighbors." (Wolfram)

A particularly interesting pattern can be saved on the Data Disk for recall at a later time.

Saving Your Results

```
      1-2
0 1 2 3 4 5 6 7 8 9 = SUM
0 1 0 2 0 3 0 1 0 0 = NEW CELL VALUE
C=CONTINUE S=SAVE
```

When the above lines appear at the bottom of the screen, pressing "S" prompts you to put the Data Disk in the correct drive and then asks for a name the pattern.

PLACE DATA DISK IN DRIVE 2

FILE NAME: ?

The pattern may be displayed at a later time by selecting option 2 from the Main Menu. The following screen will appear.

Recalling Your Results

```
*** DATA ON THIS DISK ***

1: 1 ON 1
2: 1-2-1

1) RECALL A FILE
2) DELETE A FILE
3) RETURN TO LIFE MENU

YOUR SELECTION :
```

At this time you may recall or delete patterns stored on the Data Disk.

PROGRAM DISK MUST BE IN DRIVE 1

After your work with the Data Disk is completed, you must be sure the Program Disk is in Drive 1.

Why Pursue Study in This Area?

Why is there interest in this field?

"Arrays of locally interacting cells are seen as potentially useful models of physical system, ranging from snowflakes to ferromagnets to galaxies. They may also have applications to questions in computer science, both practical (How should one organize a network of many interacting computers?) and theoretical (What is the ultimate limit to the power of a computing machine?). Perhaps most intriguing, the cellular automaton can be viewed as a 'digital universe' worth exploring for its own sake, quite apart from its utility as a model of the real world." (Hayes)

DNA in chromosomes is one practical example to consider. How can the order of molecules on a few strands of DNA contain all the information needed to create a human or an elephant? Perhaps a simple rule produces complex organisms from the information of the DNA molecules in the same way a rule produces complex patterns from the starting cell patterns in linear cellular automata.

The Game of 'Life'

"Life is an infinite two-dimensional lattice of square cells whose states are influenced by the states of neighboring cells. Time is also discrete and from one tick of a cosmic clock to the next, each cell is either alive or dead depending on a set of very simple rules:

- 1) If a cell is dead at time t , it comes alive at time $t + 1$ if, and only if, exactly three of its eight neighbors are alive at time t .
- 2) If a cell is alive at time t , it dies at time $t + 1$ if, and only if, fewer than two or more than three neighbors are alive at time t ." (Dewdney)

To help make 'Life' more understandable, let's work through an example.

```
*****
*
*          GAME OF LIFE          *
*
*      1)  CREATE STARTING PATTERN  *
*
*      2)  RECALL STARTING PATTERN  *
*
*      3)  DESCRIPTION              *
*
*      4)  RETURN TO DISK CONTENTS  *
*
*****
1, 2, 3, OR 4
```

From the Main Menu select option 1 to create a starting pattern. Use the arrow keys (or I,J,K,M) to move the cursor around the screen. Type "*" to bring a cell to life. Press the <SPACE BAR> to remove life from a cell. Type "R" to fill the screen randomly with living cells, or "O" to recall other starting screens from the Data Disk. When the screen is ready to start, press "P" to play.

P=PLAY *=LIFE -> =MOVE R=RANDOM O=OLD

The next prompt line allows you to "C" continue the game, to "P" print the screen in text to a printer, or to "S" save the screen on the Data Disk.

C=CONTINUE P=PRINT S=SAVE TIME:0

Introduction

Main Menu

Setting Initial Conditions

Detailed Example



The starting pattern above is an R Pentomino. This starting pattern has been studied extensively. We will use this to introduce 'Life.'

1	2	3
4	*	5
6	7	8

Each cell has 8 neighbors

			*	*			
		*	*				
			*				

			1	2	2	1	
		1	3	3	2	1	
		1	3	4	4	1	
		1	3	2	2		
			1	1	1		

		*	*	*			
		*					
		*	*				

		1	2	3	2	1	
		2	2	3	1	1	
		3	4	6	3	1	
		2	2	2	1		
		1	2	2	1		

			*				
		*	*				
	*			*			
		*	*				

Rule Explanation

Each cell has eight neighboring cells. The numbers in the cells to the right indicate the number of living neighbors each of the cells has. The next generation to the lower left is determined by the number of neighbors.

- 0,1 - Cell dies or remains dead.
- 2 - Remains the Same
- 3 - If dead, receives life. If alive, remains alive.
- 4,5,6,7,8 - Cell dies or remains dead

Any attempt to play 'Life' by hand quickly proves how useful the computer is in this endeavor.

Once the life forms start to change, they will continue to change until life dies out or until all the life forms are stable or repeat endlessly. Press "F" to freeze the screen, or the arrow keys to speed up or slow down the time between generations. Pressing the Escape Key <ESC> stops the game and returns you to the main menu.

"The Life screen, or plane, is a world unto itself. It has its own objects, phenomena, and physical laws. It is a window to an alternate universe.

Shimmering forms pulsate, grow, or flicker out of existence. "Gliders" slide across the screen. Life fields tend to fragment into a "constellation" of scattered geometric forms suggestive of a Miró painting.

Much of the intrigue of Life is the suspicion that there are 'living' objects in the Life universe. (John Horton) Conway adapted (John) Von Neumann's reasoning to prove that there are immense Life objects that can reproduce themselves. There is reason to believe that some self-reproducing Life objects could react to their environment, evolve into more complex 'organisms,' and even become intelligent." (Poundstone)

Part of the fun of playing Life is to look for some of the forms mentioned above. The Data Disk contains interesting starting patterns that can be loaded in using the second option on the Main menu.

Keep the Data Disk in Drive 2

DURING FILE WORK.

PRESS <SPACE BAR> TO CONTINUE...

The Evolution of the R Pentomino.

1. **
 **
 *

2. ***
 *
 **

3. *
 **
 * *
 **

4. **

 * *
 **

5. * *
 * *
 * *
 **

6. *
 ** **
 * *
 **

48. *

 ** **

 *

 * *
 * *
 * *

*** DATA ON THIS DISK ***

1: GUN
2: LINES
3: SHIPS
4: PULSARS
5: PENTOMINOS
6: PULSARS 2
7: HERSCHEL

1) RECALL A FILE
2) DELETE A FILE
3) RETURN TO LIFE MENU

YOUR SELECTION :

Graphs

Before examining graphs that display chaos, first look at simple exponential growth. Select option 1 from the menu below.

```
*****  
*                                     *  
*                               GRAPHS *  
*                                     *  
*      1) EXPONENTIAL GROWTH GRAPH *  
*      2) LOGISTIC GRAPH           *  
*      3) LOGISTIC BIFURCATION    *  
*      4) LOGISTIC MUSIC          *  
*      5) LOGISTIC MAPPING        *  
*      6) R*SIN GRAPH             *  
*      7) R*SIN BIFURCATION       *  
*      8) DESCRIPTION              *  
*      9) RETURN TO CONTENTS      *  
*                                     *  
*****  
1, 2, 3, 4, 5, 6, 7, 8, OR 9 1
```

Main Menu

In this example, we graph the value of a bank account over a period of 80 years. The starting balance is \$0.50, and the interest rate is 7% per year. The bank balance remains \$0.50 until the end of the first year, when the bank pays the interest. The interest is $.07 * .5$ or \$0.035. After 1 year the total in the bank account has grown to \$0.535, an increase of 3.5 cents. Another method of computing the total in the account after 1 year is to multiply the original amount (\$0.50) by 1.07 (1 plus the interest factor). After 2 years the account contains $1.07 * 0.535 = \$0.57245$. This leads to the general formula:

Exponential Growth

$$X(N+1) = R * X(N)$$

$X(N)$ = Starting value of X (\$0.50 in bank)

R = Growth factor (1.07)

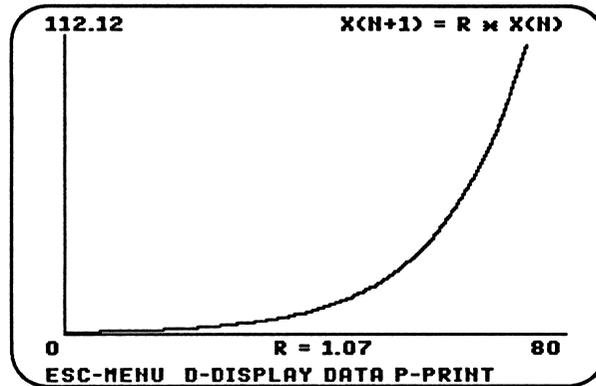
$X(N+1)$ = Amount in bank next year (\$0.535)

```
STANDARD EXPONENTIAL GROWTH  
X(N+1) = R * X(N)  
ENTER R (-1.3 TO +1.3) : 1.07  
ENTER X(0) (.1 TO 100) : .5  
PLOT 80 ITERATIONS STARTING WITH # : 0  
CHOOSE FROM 0 TO 50
```

Graph Parameters

Exponential Growth Graph

This method of computing the growth of a quantity in discrete steps is called iteration. This process could be repeated 80 times to find the amount in the bank after 80 years. The graph below shows how the balance grows to more than \$100 in 80 years.



The same formula may be used to study exponential population growth or radioactive decay. Experimenting with various R and X0 values will produce a small variety of graph shapes: increasing, decreasing, or constant. A negative value of R will force the graph to bounce between positive and negative values. The shape of the graphs can be predicted accurately by comparing the value of R to 1. This is NOT chaos.

Logistics Equation

The logistics equation can be used to examine the growth of a fish population in a limited area. The equation adds a "Death Factor" to the exponential growth equation described above:

$$X(N+1) = R * X(N) * (1 - X(N))$$

X(N) = Starting value of X

R = Growth factor

(1 - X(N)) = "Death Factor"

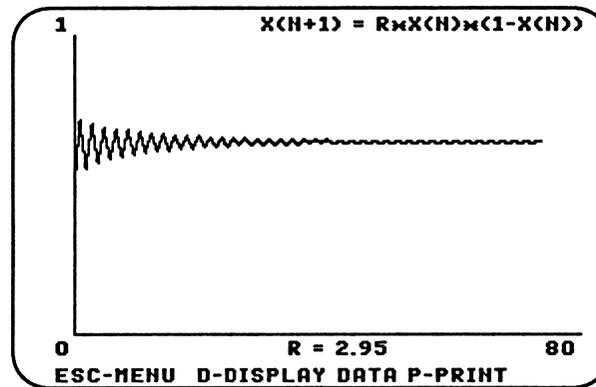
X(N+1) = X value after 1 more time period

The value of X must be positive AND less than 1. The growth factor R is less than 4. If X(N) is small, the value of the death factor (1 - X(N)) is close to 1 and it has little effect on the population. As X(N) increases, the death factor has a smaller value and can decrease the next X value significantly. Overpopulation one year reduces the population the next year.

Logistics Equation

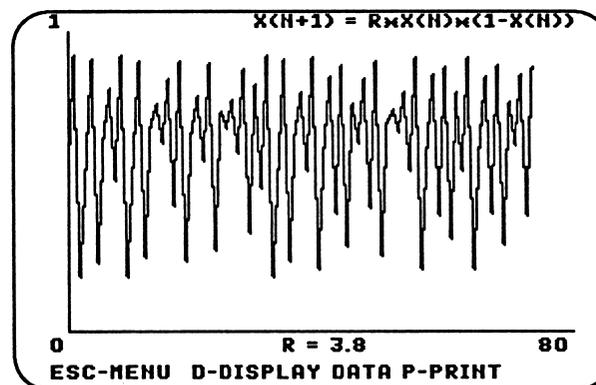
For many values of R, the graphs are fairly predictable. For example, the graph below indicates that for a starting population of .5 and a growth rate of 2.95, the fish population shows some oscillations, but eventually levels off to a constant population. At this population level, the death factor just balances the growth factor.

```
LOGISTICS EQUATION - POPULATION GROWTH
X(N+1) = R*X(N)*(1-X(N))
ENTER R (.0001 TO +4) : 2.95
ENTER X(0) (.0001 TO .99) : .5
PLOT ITERATIONS STARTING WITH # 0
CHOOSE FROM 0 TO 1000
```



The next graph shows **CHAOS**. The R value of 3.8 produces a graph that shows unpredictable variations.

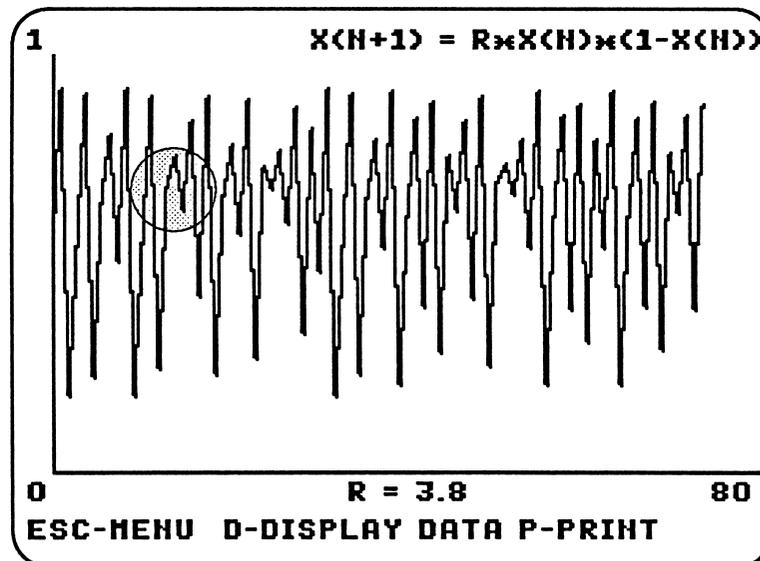
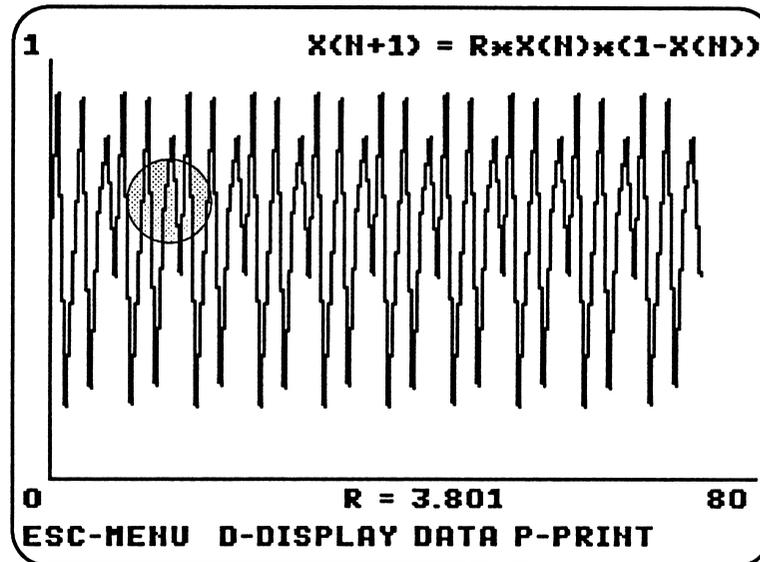
CHAOS!



```
ENTER R (.0001 TO +4) : 3.8
ENTER X(0) (.0001 TO .99) : .5
PLOT ITERATIONS STARTING WITH # 0
CHOOSE FROM 0 TO 1000
```

Effect of Small Changes in R

An R value of 3.8 will ALWAYS produce the same graph. An R of 3.801 will produce a graph with significantly different values. Examine the shaded areas.



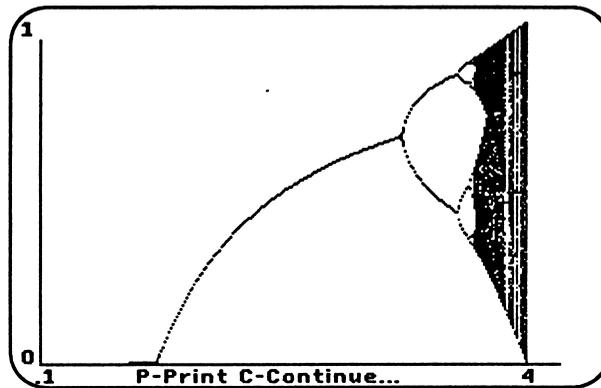
A change of .001 in R produces completely different values after only 20 iterations.

Chaos Characteristics

"Chaos is characterized by sensitive dependence on the initial conditions. Solutions are uniquely determined by initial conditions. Small changes in initial conditions are amplified exponentially. Finite precision information about initial conditions (leads to) NO finite precision information at later times." (Yorke)

Bifurcation Diagram

A bifurcation diagram summarizes the values produced for different values of R (along the horizontal axis.) The first 250 values of X(N) are ignored. The next 250 values are then plotted on a single vertical line above an R value. If the equation levels off to a single value, only one point will appear on the graph. For some values of R, the equation oscillates between 2 or more values. If R is greater than 3.7, the graph MAY be chaotic and bounce among many different values. Values above 4 produce values that increase indefinitely.



BIFURCATION DIAGRAM

$$X(N+1) = R * X(N) * (1 - X(N))$$

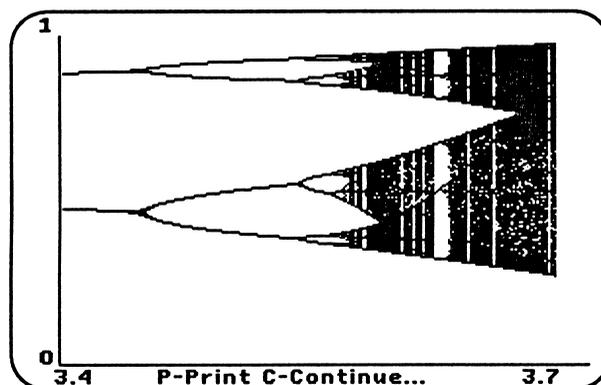
ENTER STARTING R (.0001 TO +4) : .1

ENTER ENDING R (3.3 TO +4) : 4

ENTER SPEED (1, 2 OR 3) : 1

1-SLOW, MORE DETAIL 3-FAST, LESS DETAIL

A close look at the graph shows that the number of points go from 1 to 2 to 4 to 8 to 16... possible values for adjacent values of R. Sometimes the graph will show that in the middle of the chaos, the equation will suddenly show only 3 values. These windows can be examined by choosing the graph coordinates carefully.



Expanded Graph

Musical Chaos

Option 5 will play notes along with the graph. Sometimes patterns in the graphs are more noticeable through the tones produced.

ENTER NOTE DURATION : 200
CHOOSE FROM 1 TO 255

Logistics Mapping

Three other types of graphs help show how the logistic equation changes from time period to time period. The horizontal axis is the current value of $X(N)$, the vertical axis is the next value of X , $X(N+1)$. If DOTS is selected, only the points are plotted. If LINES is selected, a straight line is drawn between the successive points. If COBWEB is selected, a line is drawn from the point to the diagonal, and then on to the next point. Different values of R will produce drastically different graphs.

LOGISTICS MAPPING - $X(N+1)$ VS $X(N)$

$$X(N+1) = R \cdot X(N) \cdot (1 - X(N))$$

ENTER R (.0001 TO +4) : 3.8

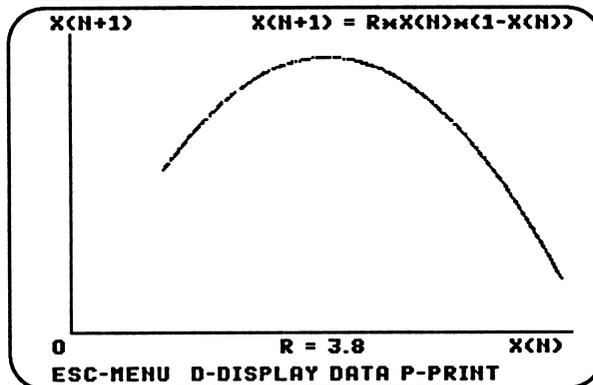
ENTER X(0) (.0001 TO .99) : .5

PLOT ITERATIONS STARTING WITH # 0
CHOOSE FROM 0 TO 1000

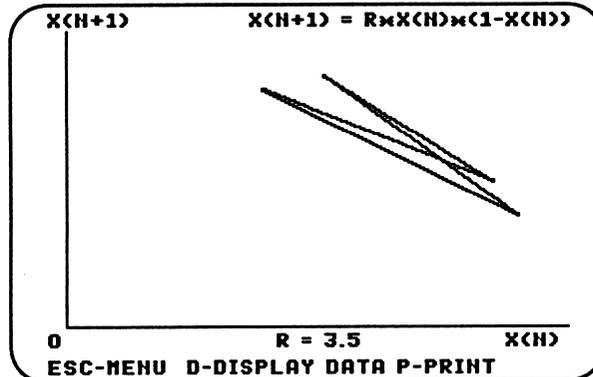
GRAPH FORM : 1) DOTS - 1000
2) LINES - 100
3) COB WEBS - 100

ENTER SELECTION : 2

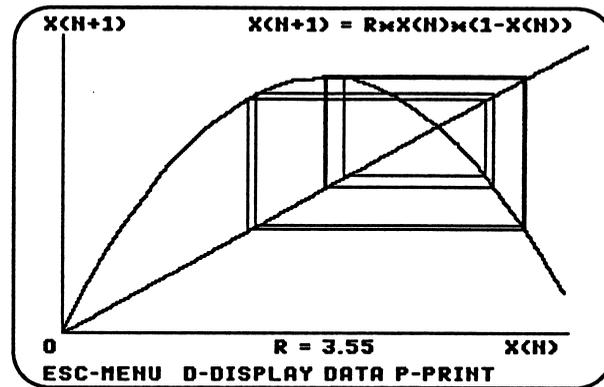
Dots



Lines



Cobwebs



The previous graphs demonstrate how a new point (y value) is related to the point just before it (x value). All the points will appear on the inverted parabola, as shown on the "Dots" graph. The number of points and their distribution depend on the value of R. Both the "Lines" and the "Cobwebs" graphs help you trace the progression of values. Values of R which produce periodic X(N)'s show the simple repeating patterns displayed on these pages. Values of R that produce chaotic graphs show much more complex patterns.

The logistics equation is only one of many equations that show chaos and bifurcation. Another equation is:

SINE FUNCTION

$$X(N+1) = R * \text{SIN}(X(N))$$

This equation may be graphed, and its bifurcation diagram plotted as before.

Why are chaotic graphs studied? What use can they be? Below are quotes from different experts.

"Chaotic nonlinear dynamics is a vigorous, rapidly expanding field. Many important future applications are to be expected in a variety of areas. In addition to its practical aspects, the field also has fundamental implications. According to Laplace, determination of the future depends only on the present state. Chaos adds a basic new aspect to this rule: small errors in our knowledge can grow exponentially with time, thus making long-term prediction of the future impossible." (Grebogi, et al)

Chaotic Sine Graphs

Why Study the Graphs?

Chaos in Many Fields

"(Early scientists) were joining a movement: chemists, ecologists, economists, climatologist now find themselves trying to reconstruct strange attractors from crude data, just as Shaw had in his dripping-faucet studies. Experts on finance use the techniques developed by the Santa Cruz group to sift through decades of daily stock market data, for example, seeking patterns they believe must be there. Many physiologists now believe that chaos provides a way of predictin—and perhaps treating—erratic rhythms in the processes that govern life, from breathing to heartbeats to brain function. At MIT, medical physicists are comparing human electrocardiograms with data from a computer model of the chaotic heart twitches in an attempt to predict well in advance when the organ may undergo fatal spasms. Some ecologists are using the mathematics of chaos to explore how, in the absence of random environmental changes, populations of species could erratically increase and decrease on their own."
(Gleick)

Attractors

"From Smale's point of view, the most important property of a dynamical system is its long-term behavior. This 'selects' a much simpler set of motions from among those of the entire system... (Engineers) talk of 'transients,' when the system is switched on, as opposed to what it settles down to if you wait a while... But for an overall view of the general nature of the system, rather than fine detail, you can ignore transients. So what does a general dynamical system do in the long run? It settles down to an *attractor*. An attractor is defined to be... whatever it settles down to! The essence of an attractor is that it is some portion of the phase space such that any point which starts nearby gets closer and closer to it." (Stewart)

Introduction

```

*****
*
*          ATTRACTORS
*
*      1) GINGERBREAD MAN
*
*      2) SIN ATTRACTOR
*
*      3) SIN ATTRACTOR II
*
*      4) HENON ATTRACTOR
*
*      5) DESCRIPTION
*
*      6) RETURN TO CONTENTS
*
*****
1, 2, 3, 4, 5, OR 6
    
```

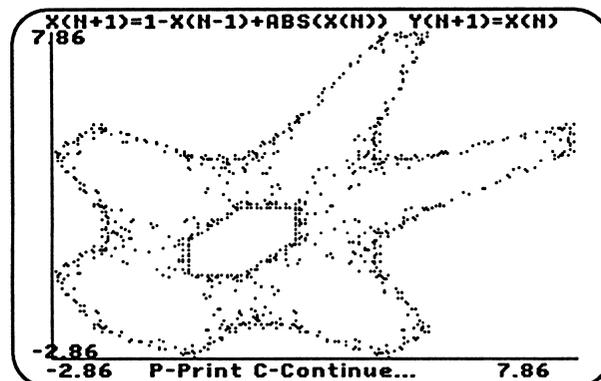
Main Menu

GINGERBREAD MAN

$$X(N+1)=1-X(N-1)+ABS(X(N)) \quad Y(N+1)=X(N)$$

ENTER X(0) VALUE (-2/ TO +2) : -.1

ENTER Y(0) VALUE (-2 TO +2) : 0



Gingerbread Man Attractor

Explanation

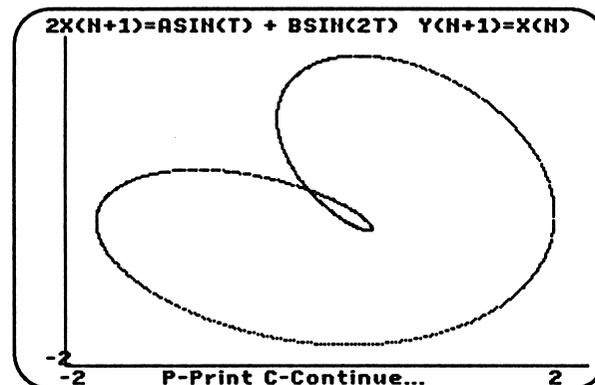
The first attractor is the "Gingerbread Man." The graph plots the old value of X on the vertical axis, and the new value of X on the horizontal axis. The new value of X depends on the two previous values of X. The graph depends on the starting values of X and Y. Experiment with different starting points to see which form the Gingerbread Man, and which produce just a few dots on the screen. Try points both inside the empty spaces of the man, as well as a variety of points within the man and outside the man.

$$X(N+1)=1-X(N-1)+ABS(X(N)) \quad Y(N+1)=X(N)$$

The graph is an attractor because initial points from different regions don't randomly produce points on the screen, but produce points within the man.

Sin Attractor

Other attractors are shown below.



ATTRACTOR

$$X(N+1)=ASIN(T) + BSIN(2T) \quad Y(N+1)=X(N)$$

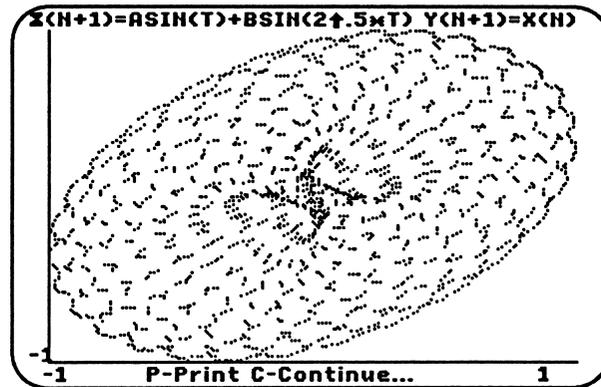
ENTER A VALUE (0 TO 2) : 1

ENTER B VALUE (0 TO 2) : 1

In the above equation, T is the series of counting numbers: 1,2,3,4... Different parameters change the size of the inner loop as well as other characteristics of the curve. Notice that the curve fills in with points that appear with no special order.

Another attractor also deals with the SIN trigonometric function. This time the square root of 2 ($2^{.5}$) is used within one of the expressions. The appearance is completely different.

Another Sin Attractor



ATTRACTOR

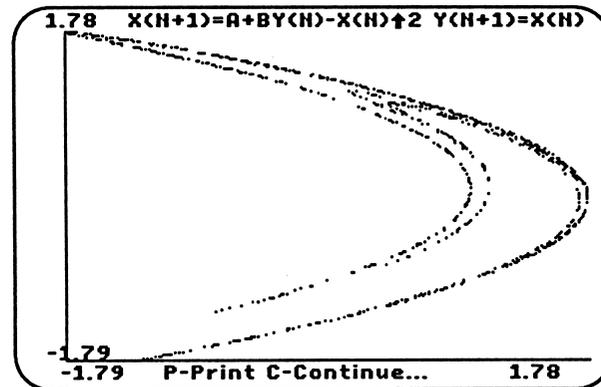
$X(N+1)=ASIN(T)+BSIN(2^{.5}T) \quad Y(N+1)=X(N)$

ENTER A VALUE (0 TO 2) : .5

ENTER B VALUE (0 TO 2) : .5

The Henon Attractor has been studied in great detail.

Henon Attractor



$X(N+1)=A+BY(N)-X(N)^2 \quad Y(N+1)=X(N)$

ENTER A VALUE (.0001 TO 2) : 1.4

ENTER B VALUE (.0001 TO 2) : .3

ENTER X(0) VALUE (-2 TO +2) : 0

ENTER Y(0) VALUE (-2 TO +2) : 0

Why Study Attractors

Why study attractors?

"In 1968, Michel Henon of the Institute of Astrophysics in Paris proposed a simple quadratic mapping of the plane as a model for the study of dynamical systems such as the motion of asteroids, satellites, or charged particles in an accelerator... Although the mapping Henon proposed is easy to describe and program, it yields results of great complexity. Since Henon mappings simulate the behavior of physical systems, they indicate that many such systems are more complex than previously imagined." (Hughes)

Attractors in History

"Human history is the ultimate chaotic system. No theory of history can predict which empires will rise and which will fall, only that some will rise and some will fall. Nor can it predict which religions or philosophies will have influence, only that some religions and some philosophies will have influence. Empires, religions, and philosophies are the norms of human civilization-the "strange attractors" of chaos. Stir the world up however you will, and in time it settles down with new empires, religions, and philosophies. You just can't predict what they will be." (Cromer)

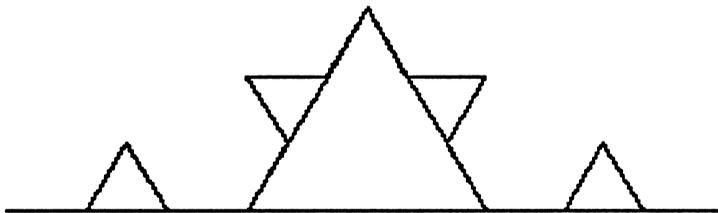
Fractal Lines

Introduction

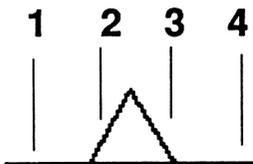
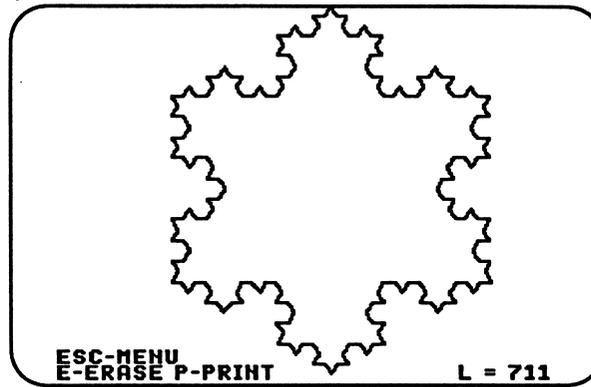
"What are fractals? How are they different from Euclidean Shapes?...First fractals are a decidedly modern invention. Although possessed of turn of the century "pathological" ancestors, they have been recognized as useful to natural scientists only over the last 10 years (since '78.) Second, whereas Euclidean shapes have one, or at most a few, characteristic sizes or length scales (the radius of a sphere, the side of a cube), fractals, like a coastline, possess no characteristic sizes. Fractal shapes are said to be self-similar and independent of scale of scaling. Third, Euclidean geometry provides concise accurate descriptions of man-made objects but is inappropriate for natural shapes. It yields cumbersome and inaccurate descriptions. It is likely that this limitation of our traditional language of shape is at least partly responsible for the striking qualitative difference between mass produced objects and natural shapes. Machine shops are essentially Euclidean factories: objects easily described are easily built. Fractals, on the other hand, provide an excellent description of many natural shapes and have already given computer imagery a natural flavor. Finally, whereas Euclidean shapes are usually described by a simple algebraic formula (E.g. $r^2 = x^2 + y^2$ defines a circle of radius r), fractals, in general, are the result of a construction procedure or algorithm that is often recursive (repeated over and over) and ideally suited to computers." (Peitgen)

One of the best known fractal shapes is Koch's Snowflake. An equilateral triangle, with each side equal to $1/3$ the length of the line segment, is placed in the center of each line segment. Next, triangles are placed in the center of the line segments of the added triangles. Below is a detailed diagram for just one side of the triangle.

Koch's Snowflake



Complete Flake



"The Koch snowflake. 'A rough but vigorous model of a coastline,' in Mandelbrot's words. To construct a Koch curve, begin with a triangle with sides of length 1. At the middle of each side, add a new triangle one-third the size, and so on. The length of the boundary is $3 * 4/3 * 4/3 * 4/3 \dots$ infinity. Yet the area remains less than the area of a circle drawn around the original triangle. Thus an infinitely long line surrounds a finite area." (Gleick)

Fractal Dimensions

One method of determining the fractal dimension of a fractal shape like the snowflake is shown below:

N = Number of identical parts added to each line segment

d = Number of divisions made in original segment to get length of added lines.

For the Koch snowflake, the original line segment is divided in 3 divisions. Each line segment in the added triangle is $1/3$ of the original line length. Therefore, $d=3$. The added triangle in the middle of the line is made up of 2 segments, but two equal line segments exist on each side of the triangle. The number of identical parts is $4 = N$.

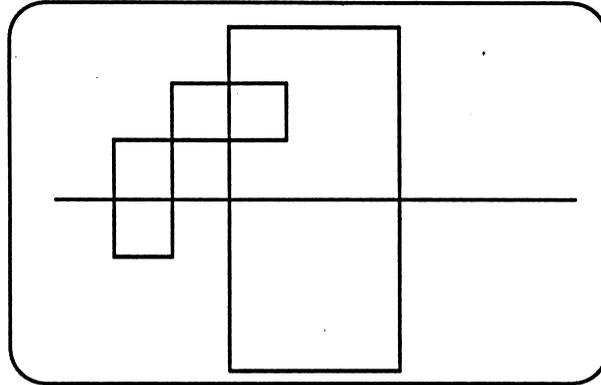
$$D = \log(N)/\log(d) \quad \text{or} \quad d^D = N$$

$$D = \log(4)/\log(3)$$

$$d = 1.26$$

The fractal dimension of the snowflake is 1.26

Peano Curve



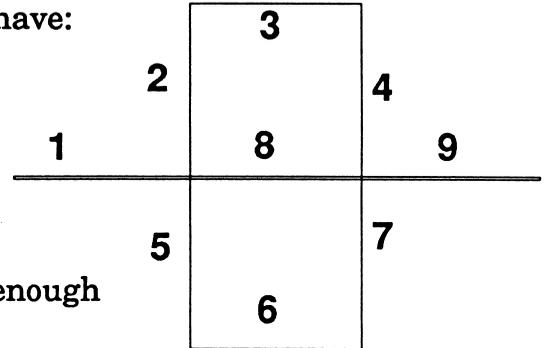
The Peano curve starts with a single line segment. That original segment is divided into thirds. The next step is to build a square above and a square below the middle of the line. This requires a total of 9 segments. Using the previous method for finding the fractal dimension, we have:

$$N = 9$$

$$d = 3$$

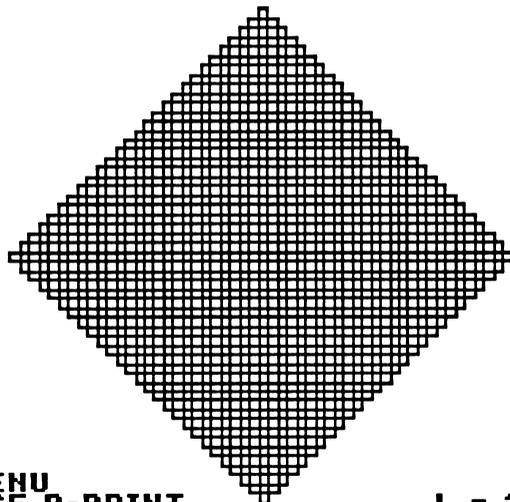
$$D = \log(9)/\log(3)$$

$$D = 2$$



The Peano curve fills up a space completely if enough rectangles are added to enough line segments.

Space Filling Curve



Main Menu

```
*****  
*  
*          FRACTAL CURVES          *  
*  
*    1) SNOWFLAKE - LINE            *  
*    2) SNOWFLAKE - CURVE          *  
*    3) SQUARE - LINE              *  
*    4) SQUARE - CURVE            *  
*    5) RECTANGLE - LINE           *  
*    6) RECTANGLE - CURVE         *  
*    7) PAENO - LINE               *  
*    8) PAENO - CURVE              *  
*    9) DESCRIPTION                *  
*   10) RETURN TO DISK CONTENTS    *  
*  
*****  
1, 2, 3, 4, 5, 6, 7, 8, 9, OR 10
```

Many different fractal lines can be selected from the menu. The "Line" selections display a single line in additional detail. The "Curve" selections show the result when the pattern is reproduced on several lines. The new line patterns can be added on to the original art work, or you can erase the original line segments and show only the added lines. Press "E" to erase the previous drawing. Press "P" to print out the High Resolution picture if printer is configured properly. Press the <Space Bar> to show the next picture, or <ESC> to return to the menu.

ESC-MENU
E-ERASE P-PRINT

L = 177

The bottom right hand corner shows the total length of the lines added on the screen. This helps indicate how quickly the length of the line grows after each operation.

Random Walks

"Is it a zinc ion, a bit of soot or a staggering drunk wandering about on the display screen? Does it join others of its kind to form a metallic flower, a dark patch or an insensible crowd? The answer depends only on one's imagination but—flower, patch or crowd—the shape of the aggregate structure is unquestionably fractal; its fractal dimension (which is between 1 and 2) can even be measured.

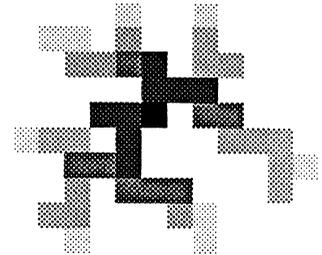
(The computer program) directs a succession of particles to wander aimlessly about the screen until they eventually encounter a collection of stationary kindred. As soon as the encounter takes place, the particle's motion is frozen. As several hundred particles accumulate one by one in this way, the collection slowly develops the irregular branches and tendrils characteristic of fractal shapes." (Dewdney)

A well known example of a random walk is Brownian Motion. Around 1827, the botanist Robert Brown studied the random motion of pollen grains he viewed through a microscope. Water molecules, which were not visible to Brown, were hitting the pollen particles from all sides at a random rate. The pollen particles moved erratically. A second example is a drunk trying to walk across an open area. Each step is a random direction. Given enough time, the drunk will reach his destination, as well as every other location in the area.

You can select several examples of random walks from the Main Menu shown below.

```
*****
*
*          RANDOM WALKS          *
*
*   1) SIMPLE RANDOM WALK      *
*   2) STICKY SINGLE DOT      *
*   3) STICKY LINE            *
*   4) STICKY CIRCLE          *
*   5) RECALL SCREEN          *
*   6) DESCRIPTION            *
*   7) RETURN TO DISK CONTENTS *
*
*****
1, 2, 3, 4, 5, 6, OR 7
```

Introduction



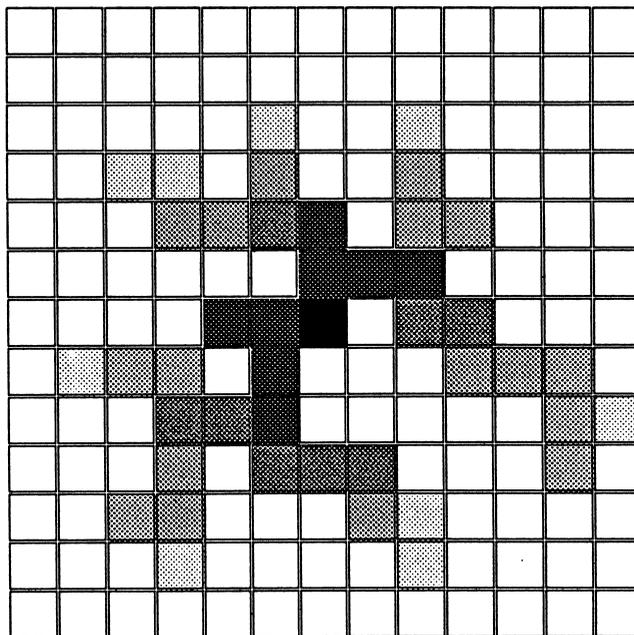
Brownian Motion

Main Menu

Sticky Particles

ESC-MENU T-TRAIL ARROWS-SPEED

If the first option is selected, the particle will move at random around the screen. If "T" is typed the particle will leave a trail around the screen. Pressing "T" again will remove the trail. The arrow keys will slow down or speed up the motion of the particle. Press the Escape Key to return to the Main Menu.



All the other options produce "sticky" particles. The particles move until they hit another stationary particle. Each of the options has a different set of initial stationary particles. The example above shows the results of particles collecting around a single stationary dot in the center of the screen.

ESC-MENU C-CONTINUE S-SAVE DOT: 350

The patterns generated can be saved on the Data Disk and recalled at a later time.

Aggregation

"Imagine growing a cluster by adding one particle at a time so that as each particle comes in contact with the growing object, it sticks and never tries another site, that is, it simply stays put. Such a process is called aggregation. It is an extreme example of a nonequilibrium process, because no rearrangement takes place at all.

Diffusion Limited Aggregation

Now suppose the particles diffuse to the cluster by means of a random walk: a sequence of steps whose magnitude and direction are determined by chance... The aggregation of particles by means of random walks is what Witten and I call diffusion-limited aggregation... The simulation suggests that diffusion-limited-aggregation (DLA) clusters are fractals.

Although DLA is simple to describe and to simulate, the process is still not well understood at a deeper level. Why, for example, does DLA give rise to fractals rather than to, say, amorphous blobs with no symmetry at all? Why are loops rarely formed? How does the fractal dimension depend on the dimension of space? Answers to these questions remain at large, and they pose a remarkable problem to the theoretical physicist because none of the ordinary mathematical tools seems to work when applied to them." (Sander)

Applesoft BASIC is fast enough to make a BASIC random walk usable. Although the program on the disk is strictly machine language, the program below will show DLA on the screen after 10 minutes or so. It is possible that bugs in the random number generator may cause the point to repeat its moves in a loop that avoids other points on the screen. You will need to press a key to stop the program if that happens.

BASIC Program

```
10 REM RANDOM WALKS
20 GR : COLOR= 7: PLOT 20,20: REM INITIAL CONDITIONS AND STATIONARY PO
INT
30 ANGLE = 2 * 3.14 * RND (9):X = INT (20 + 20 * SIN (ANGLE)):Y = INT
(20 + 20 * COS (ANG)): REM CHOSE STARTING POINT ON CIRCLE WITH 20,20 A
S CENTER
40 COLOR= INT (C / 20 + 1): REM CHANGE COLOR EVERY 20 DOTS
50 IF SCRN(X,Y) < > 0 THEN PRINT CHR$(7): GOTO 30: REM ALERT USE
R THAT STARTING POINT PART OF CLUSTER
60 XO = X:YO = Y: REM STORE PREVIOUS POSITION
70 MOVE = INT (4 * RND (9) + 1): REM CREATE RANDOM # BETWEEN 1 + 4 TO
DETERMINE NEXT MOVE
80 IF MOVE = 1 THEN X = X + 1: GOTO 200
90 IF MOVE = 2 THEN X = X - 1: GOTO 200
100 IF MOVE = 3 THEN Y = Y + 1: GOTO 200
110 IF MOVE = 4 THEN Y = Y - 1
200 IF X < 1 THEN X = 1
210 IF X > 38 THEN X = 38
220 IF Y < 1 THEN Y = 1
230 IF Y > 39 THEN Y = 39: REM KEEP DOT ON SCREEN
240 IF PEEK (- 16384) > 127 THEN END : REM CHECK FOR KEY PRESS
300 PLOT X,Y: COLOR= 0: PLOT XO,YO: REM PLOT THE NEW POINT AND ERASE T
HE OLD
310 COLOR= INT (C / 20 + 1):XO = X:YO = Y: REM RESET COLOR, STORE PRE
VIOUS X,Y VALUES
320 IF SCRN(X + 1,Y) > 0 OR SCRN(X - 1,Y) > 0 OR SCRN(X,Y + 1) >
0 OR SCRN(X,Y - 1) > 0 THEN C = C1: GOTO 30: REM IF NEXT TO DOT LEAVE
POINT SHOWING AND START NEW DOT MOVING
330 GOTO 70: REM MOVE THE SAME POINT TO A NEW LOCATION
```

Program Modifications

The program can be modified easily. Line 10 sets up a single stationary point at the start of the program. Any starting collection of points can be entered here using PLOT X,Y, HLIN X1,X2 AT Y, or VLIN Y1, Y2 AT X.

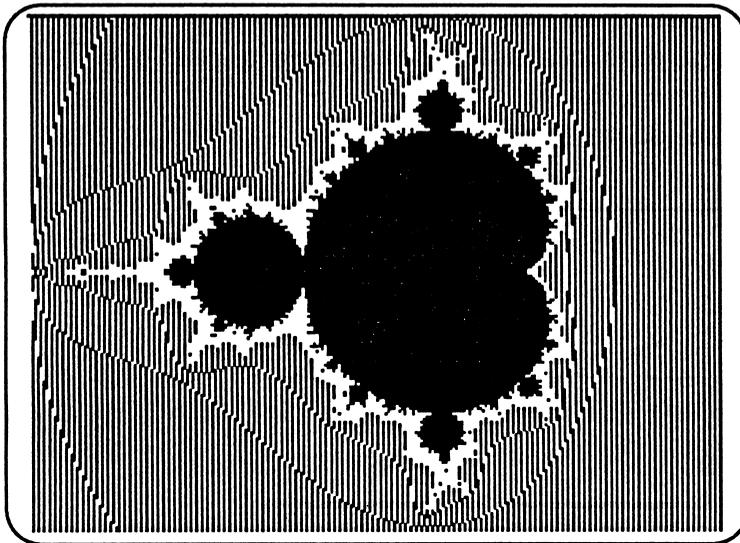
COLOR=0 in Line 300 may be changed to another number if you want a trail to show. Line 320 must be eliminated for this change to work.

It is possible to add diagonal moves, or change the rules for sticking to the cluster. Use your imagination to explore this area.

The Mandelbrot Set

"The Mandelbrot Set is the most complex object in mathematics, its admirers like to say. An eternity would not be enough time to see it all, its disks studded with prickly thorns, its spirals and filaments curling outward and around, bearing bulbous molecules that hang, infinitely variegated, like grapes on God's personal vine" (Gleick)

Introduction



Math Background

The Mandelbrot Set and related fractals work with complex numbers. There is no need to understand complex numbers to appreciate the beauty of the Mandelbrot set or to explore some of its properties. The following information is included for those of you who have the math background and curiosity to dig into the math behind the pictures.

Complex Numbers

A complex number has both a real and an imaginary part. Imaginary numbers involve "i", the square root of -1. A complex number is often written in the form of:

$$z = x + yi$$

Both x and y are real numbers, but y is the coefficient of i , and the term y is called the imaginary part of the complex number z .

When the complex number z is squared, we find:

$$\begin{aligned} z^2 &= (x + yi) * (x + yi) \\ &= x^2 + 2xyi + (yi)^2 \quad \because \text{But } i^2 = -1 \\ &= x^2 - y^2 + (2xy)i \end{aligned}$$

The magnitude of z has a form similar to the Pythagorean Theorem for distance:

$$|z| = (x^2 + y^2)^{1/2} \text{ :: } 1/2 \text{ implies square root}$$

Graph Set Up

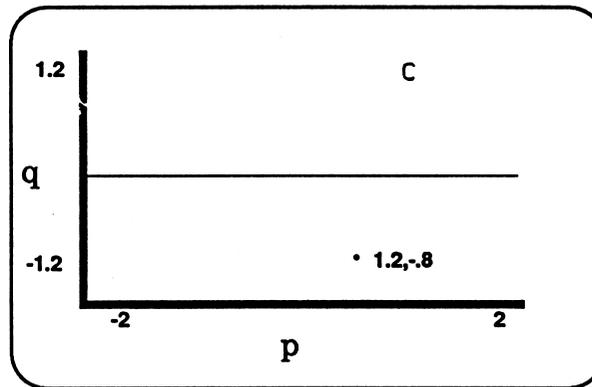
The graph of the Mandelbrot Set appears on a coordinate system like that shown below. The complex number c is used in Mandelbrot Set calculation, where:

$$c = p + qi$$

The horizontal axis locates the real (p) part of c , while the vertical axis locates the imaginary (q) part of c . The point:

$$c = 1.2 - .8i$$

is labelled on the graph.



Iteration Procedure

The Mandelbrot Set is based on the equation:

$$z_{N+1} = z_n^2 + c \text{ :: where}$$

$$z_N = x_N + y_N i \text{ and } z_1 = 0 + 0i \text{ and } c = p + qi$$

The appearance of the graph at coordinate (p,q) is determined by the effect of $c = p + qi$ on the repeated iterations of z . If the magnitude of z remains small after a large number (usually 150 or more) of iterations, the point on the graph remains black. If the magnitude of z increases in size, the color of the graph is related to the number of iterations needed for the magnitude of z to become greater than 2.

Suppose we define

$$z_N = x_N + y_N i \text{ and } c = p + qi$$

Then

$$\begin{aligned} z_{N+1} &= z_N^2 + c \\ &= (x_N + y_N i)^2 + (p + qi) \\ &= x_N^2 - y_N^2 + 2x_N y_N i + p + qi \\ z_{N+1} &= (x_N^2 - y_N^2 + p) + (2x_N y_N + q) i \\ x_{N+1} &= (x_N^2 - y_N^2 + p) \\ y_{N+1} &= (2x_N y_N + q) i \\ z_{N+1} &= x_{N+1} + y_{N+1} i \\ |z_{N+1}| &= (x_{N+1}^2 + y_{N+1}^2)^{1/2} \end{aligned}$$

If the magnitude of z_{N+1} is greater than 2, then the program stops and plots a point on the graph whose location is determined by the value of c . The color of the point is determined by N , the number of iterations to reach a value of z whose magnitude is greater than 2. If the magnitude of z is less than 2, another iteration is performed. If after 150 (a smaller number speeds up the program but loses detail) iterations the magnitude has never exceeded 2, the point is left black, and a new value of c is selected.

Applesoft BASIC Program

```

5  REM MANDELBROT SET
10 PFIRST = - 2:PLAST = 1.2:QFIRST = - 1.2:QLAST = 1.2: REM SET UP CO
    ORDINATES
15 XWIDE = 39:YHIGH = 47
20 PSTP = (PLAST - PFIRST) / XWIDE:QSTP = (QLAST - QFIRST) / YHIGH: REM
    SET UP STEPS
25 GR : POKE 49234,0: CALL - 1998: REM SET UP LO RES GRAPHICS
30 FOR HO = 0 TO XWIDE:P = PFIRST + HO * PSTP: REM SET UP HORIZONTAL C
    ORDINATES
40 FOR VE = 0 TO YHIGH:Q = QFIRST + VE * QSTP: REM SET UP VERTICAL COM
    PONENTS
50 X = 0:Y = 0: COLOR= 0:NU = 0: REM INTIAL VALUES OF X AND Y, AND COLO
    R
60 FOR N = 1 TO 150
70 XN = X ^ 2 - Y ^ 2 + P
80 YN = 2 * X * Y + Q
90 IF SQR (XN ^ 2 + YN ^ 2) > 2 THEN NU = N:N = 150: REM END LOOP AND
    SET COLOR FLAG IF MAGNITUDE > 2
95 X = XN:Y = YN
100 NEXT N
130 IF NU > 100 THEN COLOR= 6: GOTO 200
140 IF NU > 50 THEN COLOR= 5: GOTO 200
150 IF NU > 9 THEN COLOR= 4: GOTO 200
160 IF NU > 5 THEN COLOR= 3: GOTO 200
170 IF NU > 3 THEN COLOR= 2: GOTO 200
180 IF NU > 1 THEN COLOR= 1: GOTO 200
190 IF NU = 1 THEN COLOR= 15
200 PLOT HO,VE
210 NEXT VE
220 NEXT HO
230 END

```

Speeding Up the Program

The program is too slow for most practical purposes. Floating point calculations require massive amounts of time in Applesoft BASIC. Square roots and squares take more time than multiplications, and multiplications take much more time than additions and subtractions. The following substitutions speed up the program.

Line 70: $(X + Y) * (X - Y) = X * X - Y * Y = X^2 - Y^2$

Two squares and one subtraction are replaced by one addition, one subtraction, and only one multiplication.

Line 80: $XY = X * Y : YN = XY + XY + Q = 2 * X * Y + Q$

Two multiplications and one addition are replaced by only one multiplication, one variable assignment, and two additions.

Line 90: $IF (XN * XN + YN * YN) > 4 = IF SQR (XN ^ 2 + YN ^ 2) > 2$

By squaring both sides of the inequality, the square root function is eliminated.

These simple changes save only 100ths or just 1,000ths of a second each time the lines are executed, but the Low Resolution screen is $40 * 48 = 1920$ squares. The math loop is repeated a maximum of 35 times. If the average execution is 20 loops, then a full screen requires 40,000 loops. With this number of loops, even the smallest time reduction is important.

Fast BASIC Program

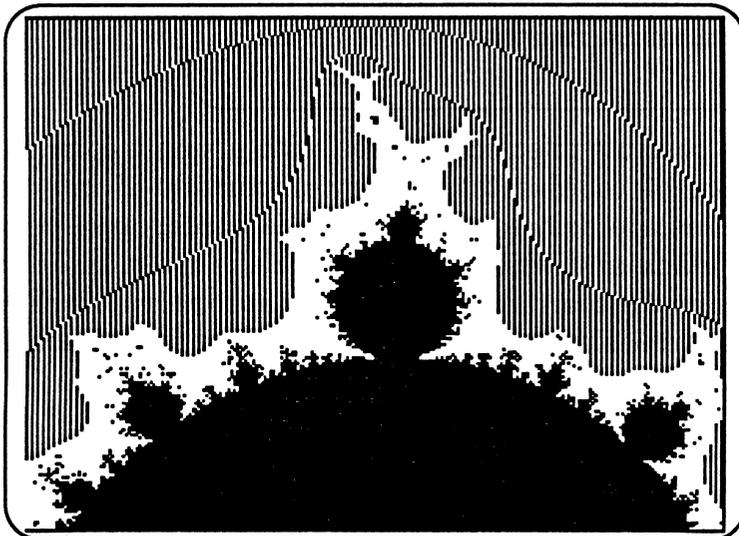
```
5 REM MANDELBROT SET
10 PFIRST = - 2:PLAST = 1.2:QFIRST = - 1.2:QLAST = 1.2: REM SET UP CO
  ORDINATES
15 XWIDE = 39:YHIGH = 47
20 PSTP = (PLAST - PFIRST) / XWIDE:QSTP = (QLAST - QFIRST) / YHIGH: REM
  SET UP STEPS
25 GR : POKE 49234,0: CALL - 1998: REM SET UP LO RES GRAPHICS
30 FOR HO = 0 TO XWIDE:P = PFIRST + HO * PSTP: REM SET UP HORIZONTAL C
  ORDINATES
40 FOR VE = 0 TO YHIGH:Q = QFIRST + VE * QSTP: REM SET UP VERTICAL COM
  PONENTS
50 X = 0:Y = 0: COLOR= 0:NU = 0: REM INTIAL VALUES OF X AND Y, AND COLO
  R
60 FOR N = 1 TO 150
70 XN = (X + Y) * (X - Y) + P
80 XY = X * Y:YN = XY + XY + Q
90 IF (XN * XN + YN * YN) > 4 THEN NU = N:N = 150: REM END LOOP AND SE
  T COLOR FLAG IF MAGNITUDE > 2
95 X = XN:Y = YN
100 NEXT N
130 IF NU > 100 THEN COLOR= 6: GOTO 200
140 IF NU > 50 THEN COLOR= 5: GOTO 200
150 IF NU > 9 THEN COLOR= 4: GOTO 200
160 IF NU > 5 THEN COLOR= 3: GOTO 200
170 IF NU > 3 THEN COLOR= 2: GOTO 200
180 IF NU > 1 THEN COLOR= 1: GOTO 200
190 IF NU = 1 THEN COLOR= 15
200 PLOT HO,VE
210 NEXT VE
220 NEXT HO
230 END
```

High Resolution Program

High Resolution graphics provides much more detail. Unfortunately, a full Hi-Res screen requires about $192*278*100 = 5,337,600$ math loops. The time needed to run the program below can require 12 or more hours, depending on the choice of coordinates.

```
5 REM MANDELBROT SET
10 PFIRST = - 2:PLAST = 1.2:QFIRST = - 1.2:QLAST = 1.2: REM SET UP CO
  ORDINATES
15 XWIDE = 279:YHIGH = 191
20 PSTP = (PLAST - PFIRST) / XWIDE:QSTP = (QLAST - QFIRST) / YHIGH: REM
  SET UP STEPS
25 HGR : POKE 49234,0: CALL - 1998: REM SET UP LO RES GRAPHICS
30 FOR HO = 0 TO XWIDE:P = PFIRST + HO * PSTP: REM SET UP HORIZONTAL C
  ORDINATES
40 FOR VE = 0 TO YHIGH:Q = QFIRST + VE * QSTP: REM SET UP VERTICAL COM
  PONENTS
50 X = 0:Y = 0: HCOLOR= 0:NU = 0: REM INTIAL VALUES OF X AND Y, AND HCO
  LOR
60 FOR N = 1 TO 150
70 XN = (X + Y) * (X - Y) + P
80 XY = X * Y:YN = XY + XY + Q
90 IF (XN * XN + YN * YN) > 4 THEN NU = N:N = 150: REM END LOOP AND SE
  T HCOLOR FLAG IF MAGNITUDE > 2
95 X = XN:Y = YN
100 NEXT N
130 IF NU > 100 THEN HCOLOR= 3: GOTO 200
140 IF NU > 50 THEN HCOLOR= 2: GOTO 200
150 IF NU > 9 THEN HCOLOR= 1: GOTO 200
160 IF NU > 5 THEN HCOLOR= 6: GOTO 200
170 IF NU > 3 THEN HCOLOR= 5: GOTO 200
180 IF NU > 1 THEN HCOLOR= 2: GOTO 200
190 IF NU = 1 THEN HCOLOR= 1
200 H PLOT HO,VE
210 NEXT VE
220 NEXT HO
230 END
```

The changes to the program to accommodate High Resolution graphics are shown in italics. The Apple II screen has trouble plotting points of different colors next to each other. The selection of color values determines the amount of color distortion on the screen.



Running the Program

```
*****  
*                                     *  
*                   FRACTALS         *  
*                                     *  
*      1) LORES MANDELBROT SET       *  
*                                     *  
*      2) HIRES MANDELBROT SET       *  
*                                     *  
*      3) RECALL MANDELBROT SET      *  
*                                     *  
*      4) DESCRIPTION                 *  
*                                     *  
*      5) RETURN TO CONTENTS         *  
*                                     *  
*****  
1, 2, 3, 4, OR 5
```

The Mandelbrot Set programs on the disk have been written in machine language for the most possible speed. Unfortunately, a high resolution graph requires about 20,000,000 math operations, and well as more than 40,000 plotting and color set operations. This takes a fair amount of time even on the fastest computers. Other fractal displays on other computers use calculations performed earlier to control the display, but the programs here work from basic definitions to calculate each point before it appears on the screen.

Selecting Coordinates

Each program permits you to set the coordinates of the graph to be plotted.

```
MANDELBROT SET  
  
Z(N+1) = Z(N)^2 + C  
  
ENTER STARTING REAL (-2/+2) : -2  
ENTER ENDING REAL (-2/+2) : 1.2  
ENTER STARTING IMAGINARY (-2/+2) : -1.2  
ENTER ENDING IMAGINARY (-1.2/+2) : 1.2
```

It is possible to select an area with a single color, so the choice of coordinates is very important if interesting screens are desired. Generally the border area between the black and white portions are the most interesting. Testing the coordinates in low resolution format will show if the area is interesting before hours are spent waiting for the high resolution picture to appear.

If the Escape Key <ESC> is entered while a graph is being plotted, the plotting will stop. Pressing ESC again will return you to the Main Menu if you are plotting the graph in Low Resolution graphics. If you are plotting a High Resolution graph, a second menu will appear on the screen.

Background

```
*****
*
*           HI RES MENU           *
*
*   1) CONTINUE PLOTTING POINTS  *
*
*   2) SAVE THE GRAPH            *
*
*   3) PRINT THE GRAPH           *
*
*   4) REDISPLAY GRAPH          *
*
*   5) RETURN TO MENU           *
*
*****
1, 2, 3, 4, OR 5
```

Programming Menu

If you choose to save the graph, you must insert the Data Disk in the appropriate drive. The drive number is determined by the selection made during the initial startup of the disk.

```
Keep the Data Disk in Drive 2
DURING FILE WORK.
```

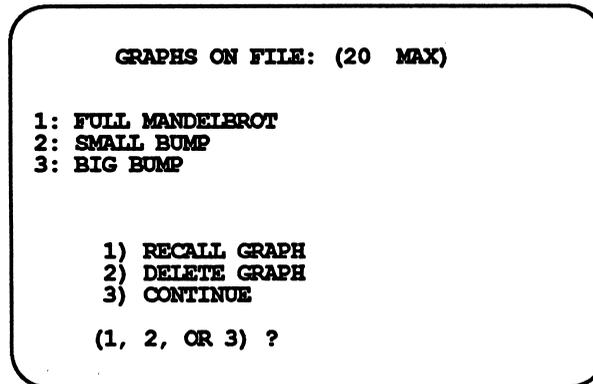
Example

If the Data Disk is NOT in the proper drive, you will receive an error message. You will be prompted to enter the name of the graph. The graph will be saved in a compressed format on the Data Disk.

You may also print the graph if the correct printer selection was made from the Program Control Panel. If this selection is not correct, the program may freeze and the graphic may be lost.

Recalling Graphs

If you want to look at graphs saved on the data disk, select **3) RECALL MANDELBROT SET** from the Main Menu. The following screen will appear.



From this menu you may recall or delete graphs from the data disk.

Helpful Hints

Examine the borderline areas between the black and white areas on the graphs. If you zoom in on a small area, fine details still exist. Unfortunately, Apple does not have the precision needed for extremely small areas, so some of the results may be caused by Apple's math problems, and not actual properties of the Mandelbrot Set.

Three Body Orbits

"The mathematicians of the 18th century ran headlong into a problem that has plagued theoretical mechanics to this day: to *set up* the equations is one thing, to *solve* them is quite another... Many problems, including basic and important ones, led to equations that could be solved... Some unanswered questions, such as the motion of **three bodies** under gravity, became notorious for their impenetrability. But somehow such equations became seen as exceptions when a more honest appraisal would have exhibited them as the rule." (Stewart)

Simple laws describe the motion of three bodies influenced by each other's gravity. However, to describe the motion accurately, the laws must know the masses of the objects, their positions, and their velocities. Small errors in measuring these parameters can cause large errors in the predicted positions of the three bodies at a later time. This program lets you test the effect of changing these parameters.

Introduction

```
*****
*
*          THREE BODY ORBITS          *
*
*  1)  CREATE & TEST ORBITS          *
*  2)  DESCRIPTION                    *
*  3)  RETURN TO DISK CONTENTS       *
*
*****
1, 2, OR 3
```

Main Menu

Select option 1 from the Main Menu. You will be asked to enter in values for the mass, position and velocity of the three objects, subject to the following restrictions.

Mass: .0001 < M < 1000
X,Y: -10 < X < +10
VX,VY: -50 < V < +50

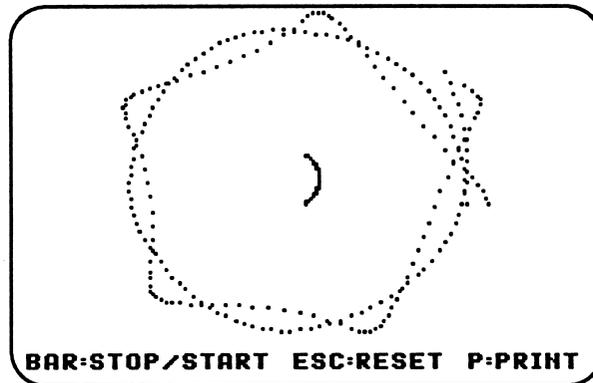
Acceptable Values

Entering Parameters

INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	250	0	0	0	0
2	10	6	0	0	45
3	1	6.8	0	0	20

The more mass an object has, the more difficult it is to influence its motion, and the more effect it has on its neighbors. The X and Y values are the horizontal and vertical coordinates of the body's position on the screen. The position 0,0 is at the center of the screen. VX and VY are the horizontal and vertical components of the body's motion. A large VY means the object is moving up the screen quickly. A large VX means the body is moving across the screen. If both VX and VY are given values, the body will move diagonally across the screen. Below is the result of the previous initial conditions.

Results



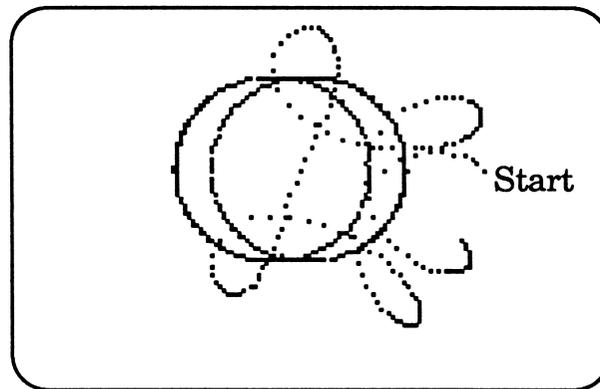
This pattern compares to the sun at the center with the moon orbiting the earth as the earth revolves around the sun. Try changing the position or velocity of the moon slightly (Suppose it were hit by a comet?) to see if the pattern changes any.

Double Sun System

Most parameters cause a collision or send a body flying off the screen. However some combinations produce amazing patterns. For example:

INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	30	-3			9
2	30	3			-9
3	.01	5		-11	11

<SPACE BAR>:PLOT <P>:PRINT <R>:REENTER



This problem might be a binary star system with a single planet. Imagine trying to predict when the sun might come up on this planet.

Let some of the orbits run for a longer period of time. Often a pattern that seems stable and seems to repeat itself will suddenly hit an alignment of the bodies that will cause a collision or fling a smaller body off to infinity.

"In human affairs, two's company and three's a divorce. In the same way, in celestial mechanics the interaction of two bodies is well behaved, but that of three is fraught with disaster. As for a dozen or more major bodies in the solar system—" (Stewart)

Conclusion

Sources

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Ott, Edward. Strange Attractors and Chaotic Motions of Dynamical Systems. *Rev. Mod. Phys.* (October 1981): 655

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Sander, Leonard M. "Fractal Growth." *Scientific American*. (January 1987): 94

Schroeder, Peter B. "Plotting the Mandelbrot Set." *Byte* (December 1986): 207

Stewart, Ian. *Does God Play Dice? The Mathematics of Chaos*. Oxford: Basil Blackwell, 1989.

Wolfram, Stephen. "Computer Software in Science and Mathematics." *Scientific American* (September 1984): 188

Yorke, James, Ott, Edward, and Grebogi, Celso. *Chaos: An Emerging Science*. Instructional Television System Outline (October 1989)

Note: Most books listed here also contain extensive reference lists.

Conclusion

Hundreds of excellent books and articles explain the concepts presented on this disk in detail. The quotes included in this manual can give you a feel for the different styles used to present chaos topics. The intent of this program was to let you explore the topics related to chaos and fractals. The disks do NOT have massive error protection or any copy protection. Feel free to make a copy of this disk, to examine the program listings and to make improvements in the code. Many programs are written to conserve space, and some routines are written in machine language. Not everything will be clear to all programmers. Speed and space conservation were priorities. Feel free to write EduTech if you have questions, problems, suggestions or comments.

In summary:

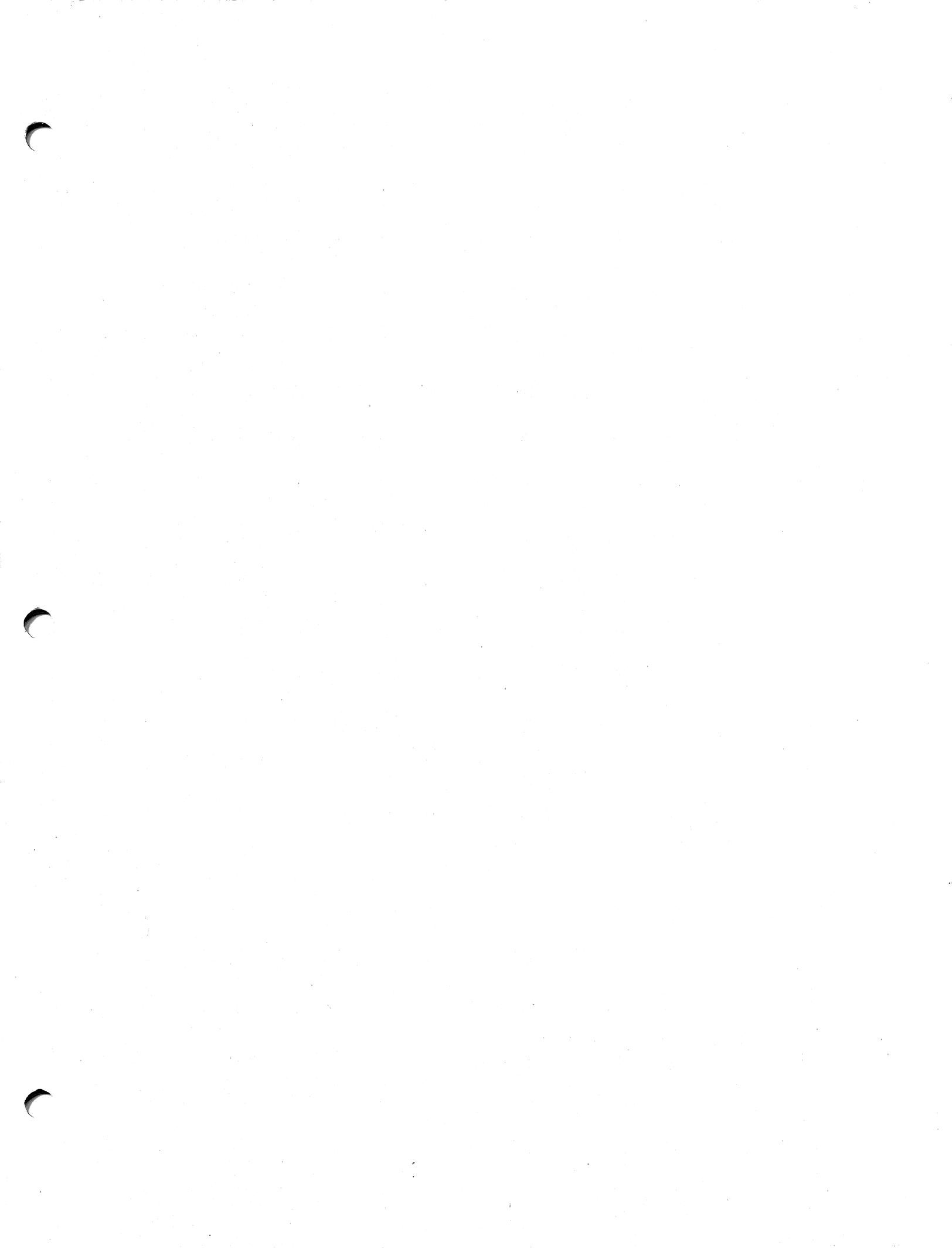
"In Newtonian mechanics, the motion of a mechanical system, such as that of the earth about the sun, is determined by mathematical equations. The solution of the equations is a relationship that describes where the objects of the system will be in the future, given their current positions. For this reason, the notion of determinism and predictability have been thought to be synonymous. With the advent of the computer, however it has been found that some systems, while changing in time in a totally determined manner, are unpredictable because they are infinitely sensitive to the conditions specified at the start. Such systems are said to be chaotic... This is happily one of those ideas that, while obscure to the mathematician, is clear to the layman... Minute changes in your life at an instant, can have profound consequences for your life's trajectory. This is what the mathematician means by chaos." (Cromer)

"Chaos has become not just theory but also method, not just a canon of beliefs but also a way of doing science. Chaos has created its own technique of using computers, a technique that does not require the vast speed of Crays and Cybers but instead favors modest terminals that allow flexible interaction. To chaos researchers, mathematics has become an experimental science, with the computer replacing laboratories full of test tubes and microscopes. Graphic images are the key. 'It's masochism for a mathematician to do without pictures,' one chaos specialist would say. 'How can they see the relationship between that motion and this? How can they develop intuition?'" (Gleick)

I hope these pictures have brought both joy and knowledge into your chaotic life.

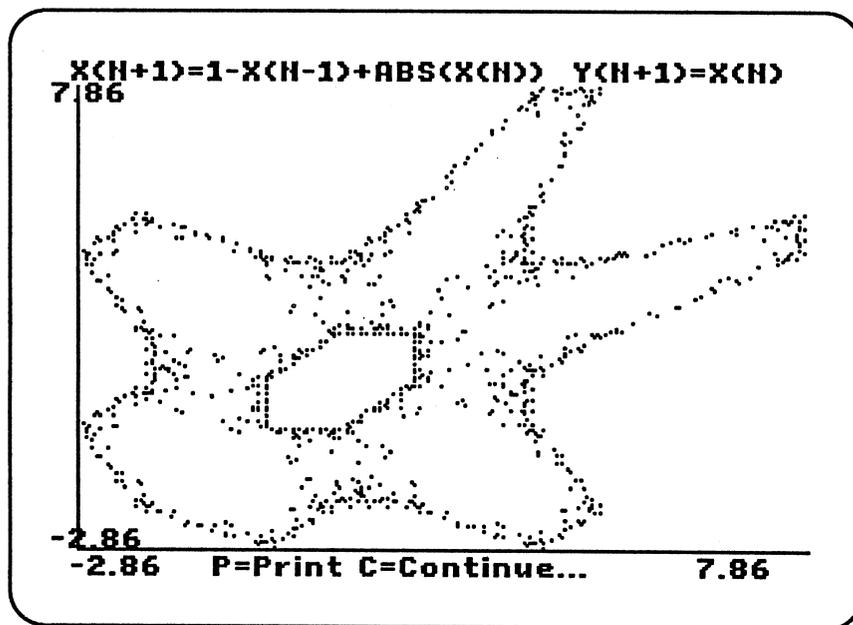
John Elberfeld

This is a blank page!



CHAOS Plus...

Workbook



EduTech 1927 Culver Road, Rochester, NY 14609
Innovative Educational Software

Chaos Workbook

This workbook is a step-by-step guide to EduTech's CHAOS Plus...disks. It explains how to use the disks in detail. It describes activities you can try that do not use the computer. The other manual and the references listed at the back of that manual supply more scientific and theoretical information.

1. Insert the MASTER DISK in Drive 1, or the internal drive.
2. Turn the monitor and the computer ON.
3. The EduTech Logo will appear, and then the Title Screen will appear.
4. Press the <ESC> key. This will bring you to the Program Control Panel. In the Control Panel, you describe the hardware setup you are using. The first time through, and when you change computer models, you must press the <ESC> key to reach the Program Control Panel. The information is stored on the disk and recalled automatically.
5. Later, when you use the program again, you will press the <SPACE BAR> to go directly to the program.
6. If you have only 1 disk drive, you must swap the MASTER DISK and the DATA DISK in and out of that single drive. Press "1" and tell the computer that the DATA DISK will be in Drive 1.
7. If you have 2 drives, put the DATA DISK in Drive 2, and tell the computer to look in Drive 2 for data. If you tell the computer to look for the DATA DISK in a drive that doesn't exist, the computer will become very confused and may stop working.

Title Screen

```

- CHAOS PLUS... -
BY
JOHN ELBERFELD

VERSION 1/90

(C) COPYRIGHT 1989: EDUTECH

<SPACE BAR> - MAIN MENU
<H> HELP          <ESC> CONTROL PANEL
    
```

Program Control Panel

```

*****
*                                     *
*          PROGRAM CONTROL PANEL      *
*                                     *
* 1) DATA DISK DRIVE : 2            *
*                                     *
* 2) GRAPHICS COMMANDS : II GS       *
*                                     *
* 3) INITIALIZE DATA DISK           *
*                                     *
* 4) LOWER CASE DISPLAY : Available   *
*                                     *
* 5) DISPLAY MAIN MENU               *
*                                     *
*****
NUMBER OF ITEM TO CHANGE : 1
    
```

Data Disk Location

```

*****
*                                     *
*          *** DATA DISK DRIVE ***   *
*                                     *
* 1) DATA DISK IN DRIVE 1           *
*                                     *
* 2) DATA DISK IN DRIVE 2           *
*                                     *
* 3) DATA DISK IN DRIVE 3           *
*                                     *
* 4) DATA DISK IN DRIVE 4           *
*                                     *
*****
SELECT DRIVE (1/4) :
    
```

Graphics Output

```
*****
*
* INTERFACE SELECTION
*
* 1) GRABBER+
* 2) SILENTYPE
* 3) PRASO
* 4) IMAGERWRITER/SUPER SERIAL
* 5) IMAGERWRITER/II C
* 6) IMAGERWRITER/II GS
* 7) CREATE PRINT COMMANDS
* 8) NO MATCH
* 9) RETURN TO CONTROL PANEL
*
*****
1, 2, 3, 4, 5, 6, 7, 8, OR 9?
```

8. If you have a printer that can print graphics, press "2." Each printer produces graphics in a different way. If you have a parallel printer, you may have a graphics interface card. If you have an ImageWriter, you must tell the program what computer it is connected to. Select the printer or its interface card from the given list.
9. Now you will your graphs on the DATA DISK that came with your program. Later, if you want to save data on your own disk, press "3" and follow the directions. The data disk will be wiped clean during initialization, so be sure to use a blank disk or an old disk with no useful data stored on it.
10. If you have an Apple II or Apple II+, you probably can not display lower case letters. Press "4" and tell the computer that lower case in NOT available.
11. Press "5" to reach the Main Menu.

Initializing a New Data Disk

Insert a blank or old disk in DRIVE 2. This program will initialize the disk and prepare it to accept data.

Press the <SPACE BAR> to INITIALIZE
or
press <ESC> to reach DISK MENU.

Lower Case Display

*** LOWER CASE ***

This sentence is mostly lower case
IF THE ABOVE SENTENCE IS NOT CLEAR,
YOU DO NOT HAVE LOWER CASE DISPLAY.

1) LOWER CASE AVAILABLE
2) LOWER CASE NOT AVAILABLE

YOUR SELECTION (1/2) :

1. When the Disk Contents Menu appears, type in the number of the activity you wish to explore. Generally, you do NOT need to press the <RETURN> key after making menu selections.
2. The rest of this manual describes how to work through each section. The sections near the front of the manual contain many details about using the programs. These details apply to most other sections and are not repeated.

```
*****
*
* DISK CONTENTS
*
* 1) CELLULAR AUTOMATA
* 2) THE GAME OF LIFE
* 3) GRAPHS OF CHAOS
* 4) ATTRACTORS
* 5) FRACTAL LINES
* 6) RANDOM WALKS
* 7) THE MANDERBROT SET
* 8) THREE BODY ORBITS
* 9) END
*
*****
1, 2, 3, 4, 5, 6, 7, 8, OR 9?
```

1. Linear Cellular Automata

1) CREATE STARTING PATTERN

1. Each horizontal line of colored squares on the screen represents a new generation of cells. Each square is called a cell. There are only 4 colors or values that a cell may have. The values are 0, 1, 2 or 3. The corresponding colors are black, white, red and green. On a monochrome monitor, you will see black, white, dark grey, and light grey.
2. First, you decide on the starting values of the first line or generation of cells. Then you enter a rule that tells how a cell will change in the next generation. Then you watch as the computer produces forty generation of cells.
3. Select "1" from the Linear Cellular Automata Menu to create a new starting pattern.
4. A flashing cursor will appear in the center of a line of dashes. Type "1 2 1" and then press return. These are the starting values for cells in this example. All the cells except for the three you typed in a number for start with a value of "0".
5. If you make a typing error, you may use the arrow keys to move the cursor back and forth across the line. Type the correct number (0, 1, 2 or 3) on top of your error.
6. *Only the values of 0, 1, 2 or 3 may be entered.*
7. Later, if you want the computer to generate a random pattern, press "R".
8. When the proper numbers are entered, press the <RETURN> key.

Linear Cellular Automata Menu

```
*****
*
*          LINEAR CELLULAR AUTOMATA          *
*      1)  CREATE STARTING PATTERN          *
*      2)  RECALL RESULTS                   *
*      3)  DESCRIPTION                      *
*      4)  RETURN TO DISK CONTENTS         *
*
*****
1, 2, 3,  OR 4
```

Creating the Starting Pattern

```
CELLULAR AUTOMATON-LINEAR FORMAT
-----121-----
Enter your starting values by typing
the numbers 1, 2 or 3 in the spaces
in the line above. A space with no
number receives a '0' value.
Use arrow keys to move down the line.
Press <RETURN> when done.
Enter 'R' to produce a random line.
```

Defining the Rule

CELLULAR AUTOMATON—LINEAR FORMAT

—121—

0 1 2 3 4 5 6 7 8 9 = SUM
 - 3 2 1 0 0 0 0 0 0 = NEW CELL VALUE

The new value of a cell depends on the sum of the values of the cell and its two immediate neighbors.

Below each sum enter 0,1,2 or 3 as the new cell value based on that sum.

Press <RETURN> when done.

Enter 'R' to produce a random line.

9. A rule tells the computer how the cell changes from one generation (line) to another. The new value (color) of the cell is shown directly below the old value. The value is found by a rule that assigns either 0, 1, 2 or 3 to the new cell. The computer adds together the value of the old cell and the values of the neighbor cells touching it on the right and on the left. Based on this SUM, the computer assigns a new value to the cell. The next screen permits you to enter the rule which controls the value of cells from generation to generation. Enter "0 3 2 1" to match the example shown here.

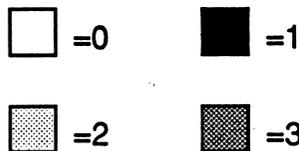
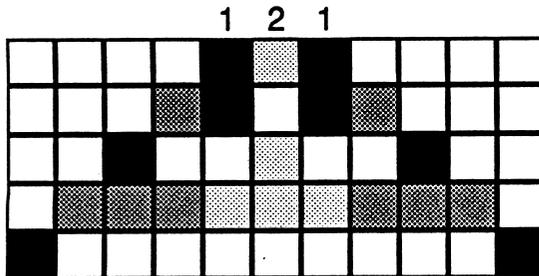
10. This rule instructs the computer to give the new cell a value of "0" if the sum of the old cell and its two neighbors is also "0". If the sum of the cell and its two neighbors is "1", the computer will assign a value of "3" to the cell. If the sum is "2", the cell is assigned "2", and if the sum is "3", the cell is assigned "1". The sums of 4,5,6,7,8 and 9 receive a value of "0".

11. Later, you may press "R" to get a random rule generated by the computer.

12. If you need to, you may press the <ESC> key to move back a screen.

13. Press the <RETURN> key when the rule is entered.

14. The following pattern will appear on the screen. It will be in color if you have a color monitor and the monitor is set to display color. If you have a IIGS, be sure the IIGS Control Panel is set to display color.



15. The top line of the Low Resolution Graphics is the color code for the starting values you entered. Since you entered "1 2 1", the first line will be black except for a white, red and white square which correspond to the "1 2 1" you typed in. Each following horizontal line represents a new generation of cells. The value or color of each new cell is controlled by the cell (and its two neighbors) above it. The value of the cells is determined by the rule you entered. The starting values and rule appears below the graph.

```

-----121-----
0 1 2 3 4 5 6 7 8 9 = SUM
0 3 2 1 0 0 0 0 0 0 = NEW CELL VALUE
C-CONTINUE S-SAVE

```

16. Press "S" to save the graphics and the starting conditions on the data disk. A maximum of 20 files of this type may be saved on the disk. You will receive the following prompts:

```

Place DATA disk in Drive 2
FILE NAME: ?TEST 2

```

17. When asked for a file name, type in a short name (under 15 letters) that helps describe the pattern you are saving.

18. If the DATA DISK is in the indicated drive, the disk will spin and your results will be saved.

2) RECALL RESULTS

1. The second option on the menu permits you to recall and delete data. Press "2" when the main menu is displayed.
2. Put the DATA DISK in the indicated drive. If the disk is not in the proper drive, or if that drive does not exist, you will lose your data and the program may restart from the beginning.

Data Disk Operations Checking for DATA DISK Location

Keep the Data Disk in Drive 2
During file work.

PRESS <SPACE BAR> TO CONTINUE...

Recall/Delete Menu

*** DATA ON THIS DISK ***

```

1: 1 ON 1
2: 312 ON 1
3: TEST 2

```

```

1) RECALL A FILE
2) DELETE A FILE
3) RETURN TO LIFE MENU

```

YOUR SELECTION :

Saving and Recalling Data

2) RECALL RESULTS Program Disk Location

PROGRAM DISK must be in Drive 1

PRESS <SPACE BAR> TO CONTINUE...

3. The names of the graphs stored on the data disk will appear at the top of the screen. The options that deal with the data files appear toward the bottom of the screen. To recall a file, press "1".
4. When prompted, enter the NUMBER of the file you want to recall:

NUMBER OF DATA FILE TO RECALL: 1

After you type in the number, you have the press <RETURN>. The computer waits to see if you are typing in more than 1 digit, like "12". The graph will appear on the screen, along with the starting pattern and the rule.

5. Press the <SPACE BAR> to continue after the graphic and its rule are displayed.

```

-----1-----
0 1 2 3 4 5 6 7 8 9 = SUM
0 1 0 0 0 0 0 0 0 0 = NEW CELL VALUE
PRESS ANY KEY...
```

6. To delete a file, press "2" when the Recall/Delete menu is on the screen. You will be asked for the number of the file you want to delete.

NUMBER OF DATA FILE TO DELETE: 3
DELETE TEST 2 (Y/N)

7. If you type a number that is not acceptable, the computer will "beep" at you when you press return. Press "Y" for YES if you really want to delete the file.
8. To return to the menu, press "3".

3) DESCRIPTION

1. For a brief description of Linear Cellular Automata, press "3" when the Main Menu is displayed. Two pages of descriptions will appear.
2. The main menu will reappear automatically.

4) RETURN TO DISK CONTENTS

1. To exit from Cellular Automata, press "4".
2. The Disk Contents Menu will appear, ready for your next choice.

On Your Own

1. You can produce an endless variety of color patterns by entering your own starting values and rules. On the left are several starting values and rules that generally produce interesting results.
2. For example, the first rule suggests you enter a "1" as the starting value for a single cell. You then enter "0 1 2 0 1 3 3 2 3 0" as the rule. The computer will produce a pattern of cells on the screen. The next two examples suggest you change the single cell starting value to "2", and then to "3". You will see significant changes in the patterns produced.
3. Press "R" to select random starting conditions, and type in the rest of the rules shown to the right. Some random starting conditions produce great results, while others just die out. Try several conditions for each rule.

Some Rules to Try

Try several different initial conditions with the rule shown below.

1

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 1 2 0 1 3 3 2 3 0 = NEW CELL VALUE
    
```

2

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 1 2 0 1 3 3 2 3 0 = NEW CELL VALUE
    
```

3

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 1 2 0 1 3 3 2 3 0 = NEW CELL VALUE
    
```

Try all the rest with random starting conditions. What numbers generate posts?

-1-2-3-11-222-

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 0 2 0 3 0 3 1 1 3 = NEW CELL VALUE
    
```

The rule below produces a constantly changing, chaotic behavior.

-22-3-213-23132323123-3-111321-33-22-23-

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 1 2 1 2 3 0 2 1 0 = NEW CELL VALUE
    
```

Notice the areas of white that appear even though different random patterns are selected for initial conditions.

-113-3-3221-11-2-2-1-2-3213233311-123-

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 2 3 1 1 2 3 0 0 3 = NEW CELL VALUE
    
```

It may take several tries to find a random initial condition that produces interesting results.

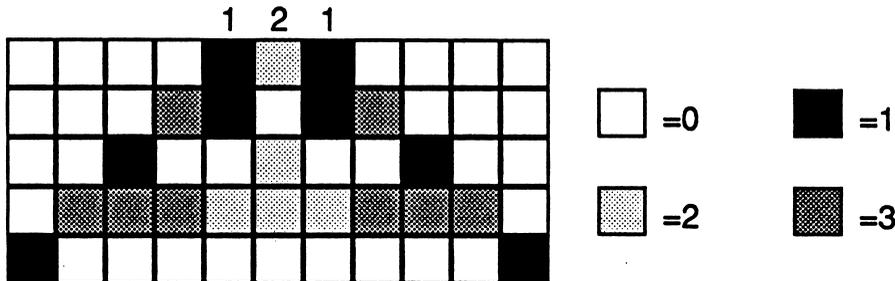
-11-331-3223-2-232-1122-1-313223233213-

```

0 1 2 3 4 5 6 7 8 9 = SUM
0 3 0 2 0 0 2 1 3 2 = NEW CELL VALUE
    
```

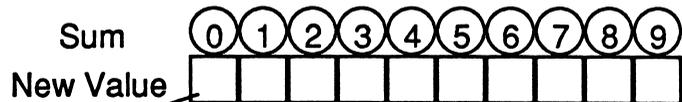

On Your Own

- Fill in the rest of the empty circles and squares. If all your work is done correctly, you will end up with the pattern shown in row five.
- Instead of displaying all the numbers, the computer uses a different color for each cell value. The computer also shows forty generations, not just the five we worked out here.



- The numbers are changed into colors or shades of grey as shown here.
- Test your understanding by creating your own rule and your own starting values. Fill the chart to the left by hand. Test your results by comparing your calculations to the computer output.

Create Your Own



- Start by deciding on a rule. Enter the rule here.
- You can enter only 0, 1, 2 or 3!
- Next enter your starting values in to top row of squares.
- Fill in the sums of the cell and its two neighbors in the circle below the cell.
- Look back at the rule figure out what numbers to put in the second row of squares.
- Fill in the rest of the circles and squares.
- Enter the starting pattern and rule into the computer, and compare the screen output to the results you found.

2) THE GAME OF LIFE

1) CREATE STARTING PATTERN

1. The Game of 'Life' is similar to Cellular Automata, except instead of working with a single line, you work with two dimensions. Instead of having a new generation on each line, the screen clears completely and a new generation appear on the screen in place of the old one.
2. Each square or cell on the screen can be either alive or dead. Dead cells are just black. Live cells are filled with a "*".
3. Press "1" to create a starting pattern. You will be told to wait a minute as the screen and memory a cleared out.
4. A blank screen full of dead cells will appear, waiting for you to tell which cells should contain life. The following prompt line will appear at the bottom of the screen:
P=PLAY *=LIFE -> =MOVE R=RANDOM O=OLD
5. Use the arrow keys to move the flashing cursor around the screen. You may also use I, J, K and M to move the cursor.
6. When the cursor is in a cell you wish to bring a cell to life, press the "*" key. To remove life from a cell, move the cursor to the cell and press the <SPACE BAR>.
7. To follow the example in this manual, type in the following life form:
*
*

8. Later, when you want to experiment with other life forms, you can press "R" to fill the screen with random life forms.
9. Later, you may look at standard life forms stored on the disk. Pressing the letter "O" will let you choose a set of life forms that have been stored on the data disk.

The Game of 'Life' Menu

```
*****
*
*          GAME OF LIFE          *
*
*      1)  CREATE STARTING PATTERN  *
*      2)  RECALL STARTING PATTERN  *
*      3)  DESCRIPTION              *
*      4)  RETURN TO DISK CONTENTS  *
*
*****
1, 2, 3, OR 4
```

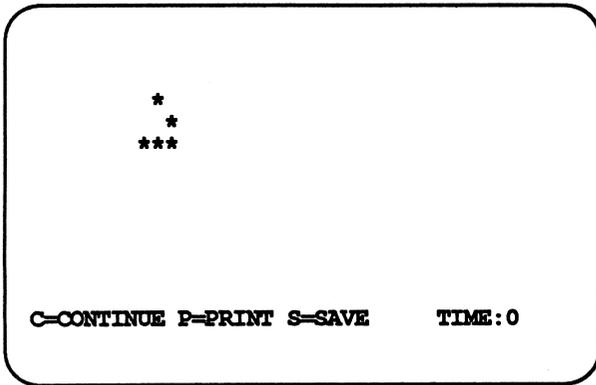
Creating the Starting Pattern

```

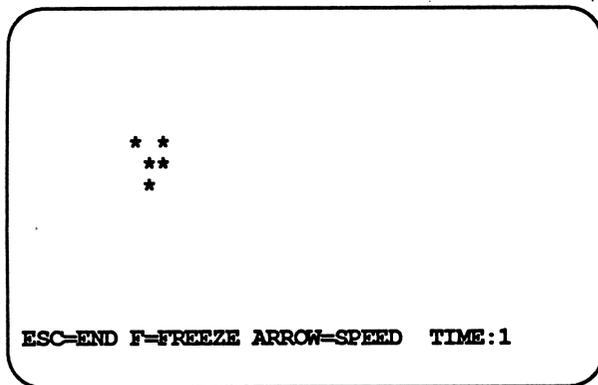
*
*
***

P=PLAY *=LIFE -> =MOVE R=RANDOM O=OLD
```

Watching the Forms Grow



Changing Screen Patterns



10. After your life form is entered, press "P" to reach the screen where you can see how the life form grows. The cells that live and die follow very strict rules based on the number of living neighbors they have.

P=PLAY *=LIFE -> =MOVE R=RANDOM O=OLD

11. Another prompt line will appear at the bottom of the screen:

C=CONTINUE P=PRINT S=SAVE TIME:0

The TIME tells you how many generations have been created up to that point.

12. Press "C" to continue. The life form on the screen will start to grow.
13. Later, you may press "S" to save the current screen displayed on the DATA DISK.
14. If you wish to print out your results at a later time, you may press "P" to print out the screen on any printer connected to Slot 1.
15. Another prompt line appears:

ESC=END F=FREEZE ARROW=SPEED TIME:1

Use the Arrow Keys to speed up or slow down the rate of growth. The speed changes one step after each generation if the arrow keys are pushed.

16. Press "F" to freeze the action on the screen.
17. Press the <ESC> key to return to the starting menu.

2) RECALL STARTING PATTERN

1. The second option on the menu permits you to recall and delete data. Press "2" when the main menu is displayed.
2. Put the DATA DISK in the indicated drive.
3. The names of the life patterns stored on the data disk will appear at the top of the screen. The options that deal with the data files appear toward the bottom of the screen. To recall a file, press "1".
4. When prompted, enter the NUMBER of the file you want to recall:

NUMBER OF DATA FILE TO RECALL: 1

The life forms will appear on the screen.

5. After the file is loaded in, you can follow the instructions as if you just typed in the life forms by hand.
6. To delete a file, press "2". You will be asked for the number of the file you want to delete.

NUMBER OF DATA FILE TO DELETE: 7
DELETE HERSCHEL (Y/N)

7. If you type a number that is not acceptable, the computer will "beep" at you when you press return. Press "Y" for YES if you really want to delete the file.
8. To return to the menu, press "3".

3) DESCRIPTION

1. For a brief description of "Life," press "3" when the Main Menu is displayed. Two pages of descriptions will appear.
2. The main menu will reappear automatically.

4) RETURN TO DISK CONTENTS

1. To exit from "Life," press "4".
2. The Disk Contents Menu will appear, ready for your next choice.

Data Disk Location

Keep the Data Disk in Drive 2
during file work.

PRESS <SPACE BAR> TO CONTINUE...

Recalling and Deleting Data

*** DATA ON THIS DISK ***

1: GUN
2: LINES
3: SHIPS
4: PULSARS
5: PENTOMINOS
6: PULSARS 2
7: HERSCHEL

1) RECALL A FILE
2) DELETE A FILE
3) RETURN TO LIFE MENU

Do It Yourself

Neighbors

1	2	3
4		5
6	7	8

The center grey cell has eight neighbors. They are numbered 1 to 8 in the pattern above.

Each cell has eight neighboring cells. Neighboring cells are cells that touch the center cell. The neighboring cells may be living or dead. The next generation is determined by the number of living neighbors each cell has.

0,1 - Cell dies or remains dead.

2 - Remains the same

3 - If dead, receives life. If alive, remains alive.

4,5,6,7,8 - Cell dies or remains dead

Living Cells

of Living Neighbors

		*			
		*			
		*			

		1	1	1	
		2	1	2	
		3	2	3	
		2	1	2	
		1	1	1	

		*	*	*	

				2	1
					1
			3	2	1

1. In this activity, you first look at the pattern of living cells in the left hand squares. Living cells are filled with "*", while dead cells are blank. In this example, you start with a vertical column of 3 living cells.

2. Next you count the number of **living** neighbors each cell has. The grid on the right leaves room for you to type in the number of neighbors for each cell. The cells that were alive have a light shade of grey in them.

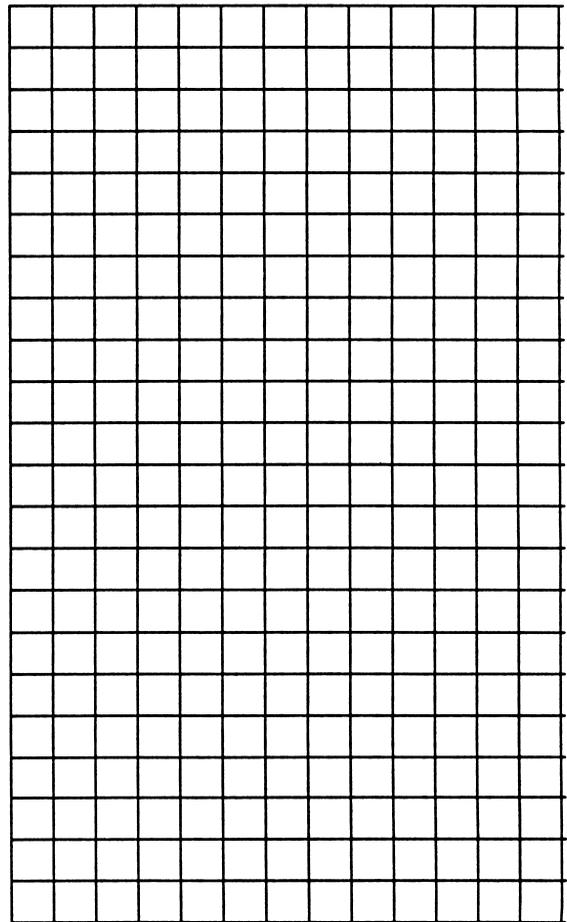
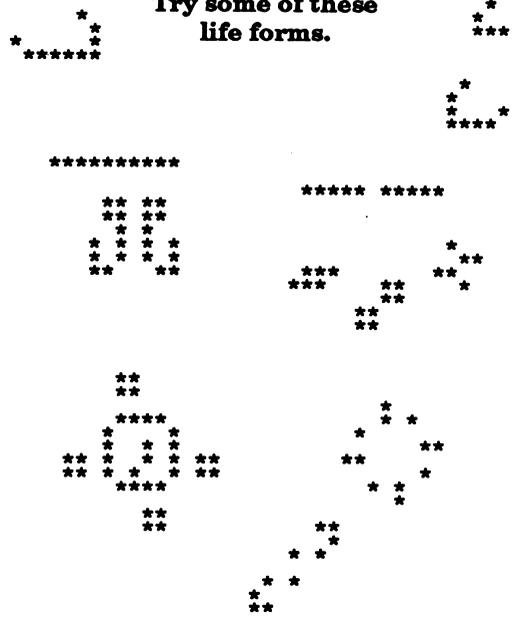
3. For example, the middle cell in the column has only two neighbors: The cell above it and the cell below it. The number "2" is entered into the center cell in the grid to the right to show the number of its neighbors.

4. Now count up the living neighbors for each cell in the grid. If there are "0" living neighbors, you can leave the cell blank. Once all the neighbors have been counted, you can determine where the life forms are for the next generation. Any cell with "3" neighbors is alive. Any living cell with "2" neighbors will be alive during the next generation. All other cells are dead.

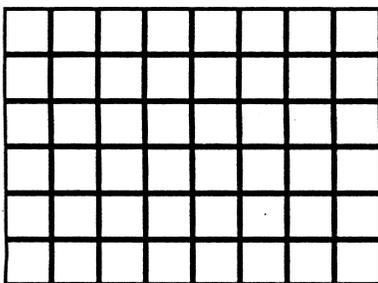
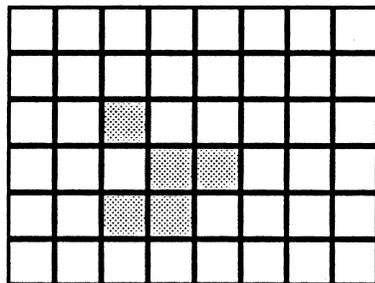
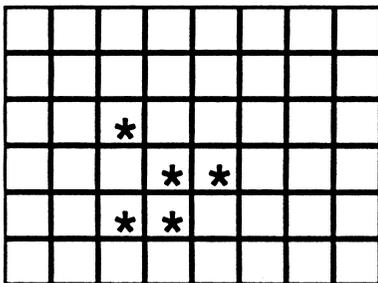
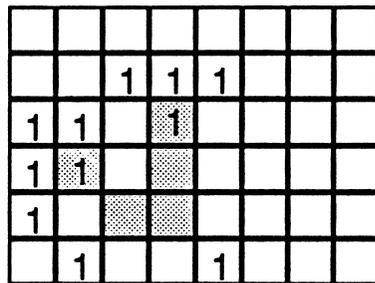
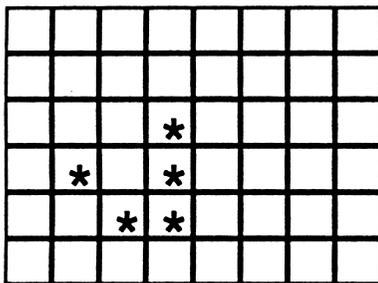
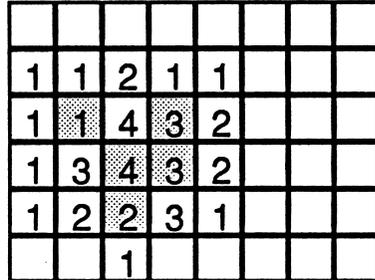
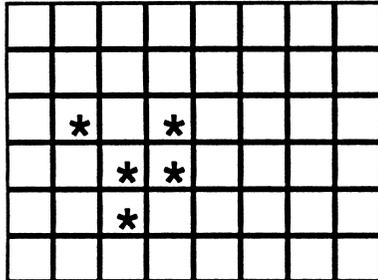
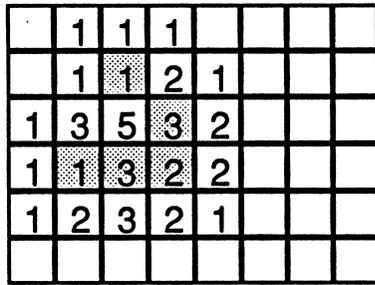
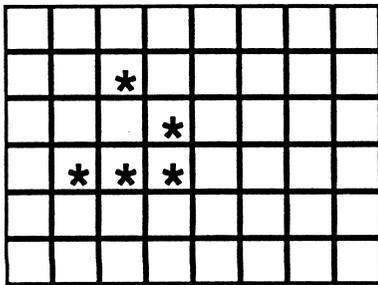
3. Fill in the rest of the missing numbers.
4. Your final life form at the bottom of the page should match your starting life form. This pattern will repeat endlessly.

5. The only cells that are alive in the second generation are three cells in a horizontal row across the center of the grid. The two end cells had 3 neighbors, and according to the rule, they received life. The center cell had only 2 neighbors, but because it was alive, it remained alive.
6. Repeat this pattern of counting up the number of neighbors for each living cell, and filling those numbers in to the right hand grid. Then apply the rule for "Life" and mark the left hand grid where the cells are alive in the next generation.
7. Your final life form at the bottom of the page should match your starting life form. This pattern will repeat endlessly.
8. Many books have been written about the various life forms that appear on the screen. Because it takes days to do what the computer does in a few seconds, it only makes sense to let the computer do most of the busy work.
9. Try entering some of the life form shown in the right column as a starting pattern. Most will produce some type of interesting result.
10. Press "R" to produce a random starting pattern and look for interesting life forms that appear naturally from a random start. You may see flashers, still life, gliders that move across the screen, and many other forms that you can name yourself. You can use the blank graph to record your results.

Try some of these life forms.



Gliders



1. The pattern below offers a bigger challenge. Follow the same procedure of entering the number of neighbors for each cell in the right hand squares.
2. Use the rules of "Life" to place "*" in the living squares in the left grid.
3. This pattern also repeats itself, but at a different position on the screen. This type of life form is called a glider. It may appear naturally when you start with a random life pattern.

3) GRAPHS OF CHAOS

2) LOGISTIC GRAPH

- This workbook will concentrate on the second option. The operation is similar for several other menu choices:
 - 1) EXPONENTIAL GROWTH GRAPH
 - 4) LOGISTIC MUSIC
 - 6) R*SIN GRAPH
- Press "2" to work with the Logistic Graph. You will have a chance to change many of the parameter (numbers) used to compute the graph.
- This equation is often used to predict population growth—usually fish. The graph shows the "population" at the end of each year.
- X(0) is the fish population at time "0". The number inside the "()" tells what year we are looking at. For example, X(5) is the fish population at the end of the fifth year. For the equation, the computer uses letters instead of all the numbers.
- X(N+1) is the population at the start of the next year. X(N) is the population at the start of the current year. The equation shows how next year's population depends on the current population. The "*" tells the computer to multiply.
- "R" is the rate of population growth. "R" may vary from .0001 (almost no growth) to 4 (4 new fish appear for every old fish). For this example, enter 3.8 for "R".
- X(0) is the starting population. It must be a decimal number less than 1. (Remember, 50¢ is \$.50—any number may be written as a decimal less than one if the units are selected properly. Also, 500,000 is really only .5 million.) Set X(0) to .5

Graphs of Chaos Main Menu

```

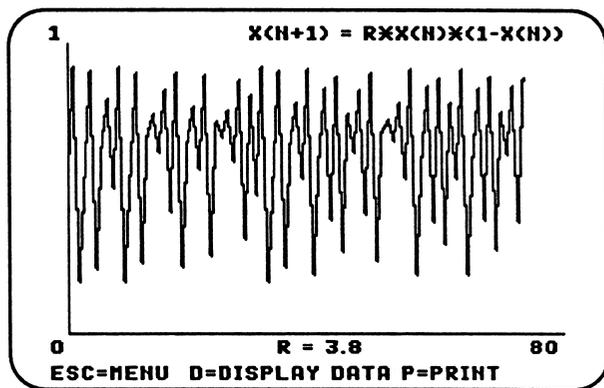
*****
*                                     *
*                               GRAPHS *
*                                     *
*      1) EXPONENTIAL GROWTH GRAPH  *
*      2) LOGISTIC GRAPH              *
*      3) LOGISTIC BIFURCATION       *
*      4) LOGISTIC MUSIC              *
*      5) LOGISTIC MAPPING           *
*      6) R*SIN GRAPH                *
*      7) R*SIN BIFURCATION          *
*      8) DESCRIPTION                *
*      9) RETURN TO CONTENTS         *
*                                     *
*****
1, 2, 3, 4, 5, 6, 7, 8, OR 9 1
    
```

Graph Parameters

```

LOGISTICS EQUATION - POPULATION GROWTH
X(N+1) = R*X(N) * (1-X(N))
ENTER R (.0001 TO +4) : 3.8
ENTER X(0) (.0001 TO .99) : .5
PLOT ITERATIONS STARTING WITH # 0
CHOOSE FROM 0 TO 1000
    
```

Results

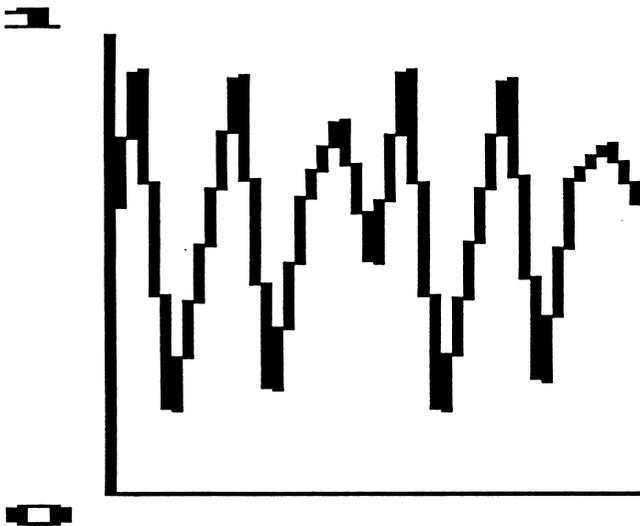


Computer Calculations

$$X(N+1) = R * X(N) * (1 - X(N)) \quad R=3.8 \quad X_0=.5$$

0: .5	20: .239629836
1: .95	21: .692388036
2: .1805	22: .809350006
3: .56209505	23: .58634978
4: .935347978	24: .921666119
5: .229794124	25: .274351201
6: .672557382	26: .756513954
7: .836851009	27: .699962247
8: .518819311	28: .798057379
9: .948654167	29: .612414835
10: .185095865	30: .901979038
11: .573174466	31: .335968841
12: .929652891	32: .847756359
13: .248513895	33: .490448955
14: .709668008	34: .949653354
15: .782949441	35: .181685071
16: .645770533	36: .564967303
17: .869253617	37: .933961148
18: .431876713	38: .234375343
19: .932365027	39: .681885458

Expanded View of Computer Graph



- The factor $(1 - X(N))$ tends to decrease the population. This factor balances the effect of population growth. It takes into account the death of fish in the pond.
- The graph can start plotting populations after many years have gone by. Enter 0 to start looking at the population from the very beginning. Later, you may enter larger numbers to see what the population is like after 100 or 500 years.

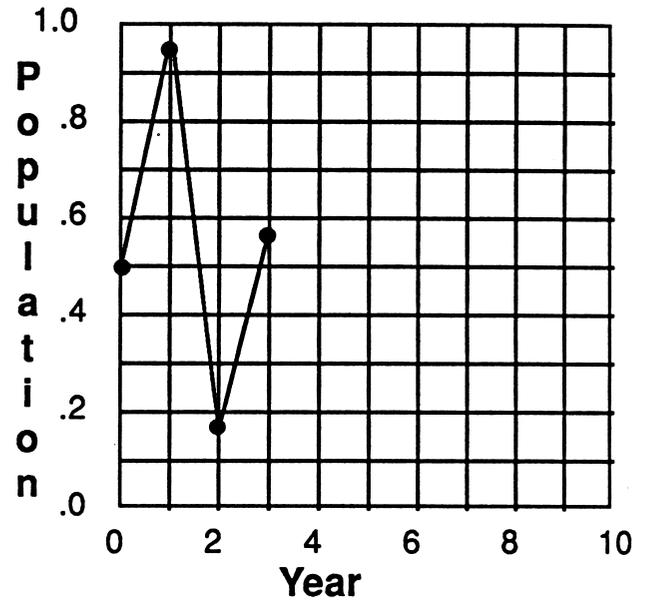
Do It Yourself

- Start with the same numbers you used on the computer:
 $R = 3.8$
 $X(0) = .5$
- $X(N+1) = R * (X(N)) * (1 - X(N))$
 The "*" is the symbol used by computers for multiplication. If you put your numbers in, you get:
 $X(1) = 3.8 * .5 * (1 - .5)$
- Calculators will save you time and trouble, but this work can be done by hand. First find the value inside the set of parentheses.
 $(1 - .5) = .5$
- $X(1) = 3.8 * .5 * .5 = .95$
- After 1 year, the population has grown from .5 to .95. Now compute the second year population.
 $X(2) = 3.8 * .95 * (1 - .95)$
 $X(2) = 3.8 * .5 * .05 = .1805$
- Similarly:
 $X(3) = 3.8 * .1805 * (1 - .1805)$
 $X(3) = 3.3 * .1805 * (.8195) = .5621$
 You can round off to 4 figures to make life a little easier. However, rounding off can have dramatic effects on your results later on. That effect is part of the Chaos experience.

Logistics Graph

8. Repeat the process until you fill in the chart below:

$X(0) = .5$
 $X(1) = .95$
 $X(2) = .1805$
 $X(3) = .5621$
 $X(4) =$
 $X(5) =$
 $X(6) =$
 $X(7) =$
 $X(8) =$
 $X(9) =$
 $X(10) =$



10. Now graph the points on the grid on the right. The first four are done for you. How does this graph compare to that on the previous page?
11. Select option 4 to produce music that matches changes on the graph:

4) LOGISTIC MUSIC

Everything is the same except you must enter in the note duration:

ENTER NOTE DURATION : 100
 CHOOSE FROM 1 TO 255

The bigger the number, the longer the note lasts.

R Value Description

More Numbers to Try

1. The value of "R" has profound effects on the appearance of the graph. Sometimes a difference of .1 or less in "R" can create a completely different graph.

Compare the **computer generated** graphs for the following values of "R":

R Value Description

.5

3.4

3.5

3.568

3.628

3.7

3.74

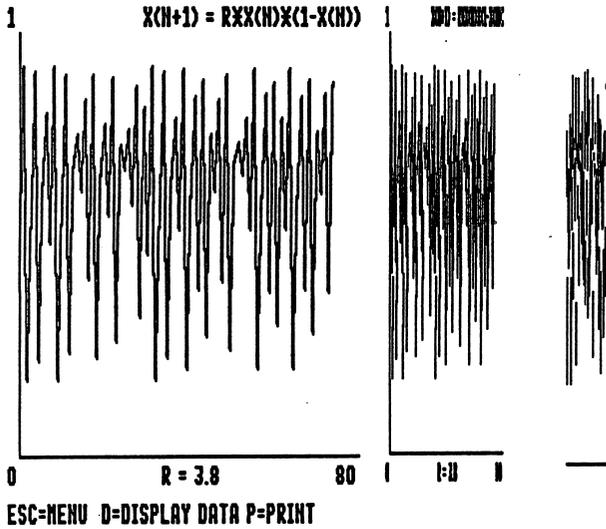
3.82

1

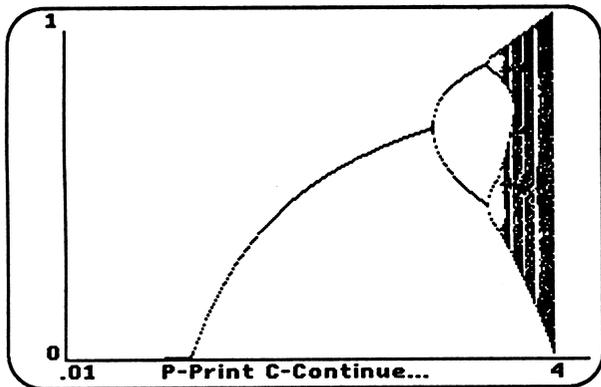
3.83

2.2

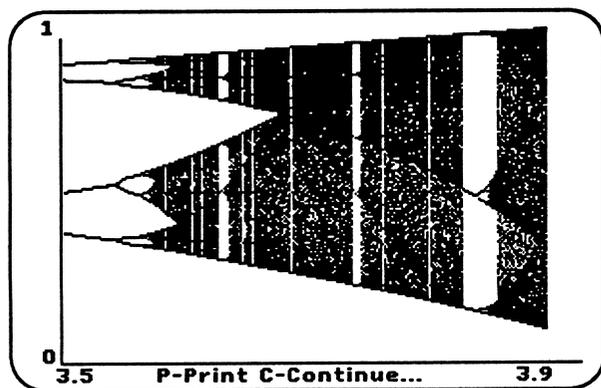
Compressing the Graph



Complete Diagram



Detailed Diagram



3) LOGISTIC BIFURCATION

1. The Bifurcation Diagram summarizes the information stored on hundreds of graphs. Instead of expanding the graphs, the diagrams compress the graphs.
2. All the points on the previous graphs are compressed into a single vertical line. The points then appear over the value of "R" on the horizontal axis.
3. The first 250 points are NOT plotted. The diagram shows only the long term values. To see the graph, enter the following values:

BIFURCATION DIAGRAM

$$X(N+1) = R * X(N) * (1 - X(N))$$

ENTER STARTING R (.0001 TO +4) : .01

ENTER ENDING R (.01 TO +4) : 4

ENTER SPEED (1, 2 OR 3) : 1
1-SLOW, MORE DETAIL 3-FAST, LESS DETAIL

Enter different starting and ending values to examine part of the graph in more detail. Choose a speed of "3" to see less detail in a shorter time.

4. Notice that for some values of "R" there are only 1, 2, or 4 points on the graph. This is **Bifurcation**. Others have hundreds of values.
5. Chaos occurs when no predictable pattern shows up on the logistics graph. The values of "R" that produce chaotic graphs are those with a nearly solid line of points above them in the bifurcation diagram. What is the lowest value of "R" that produces chaos? Do all values of "R" above that point produce chaotic graphs?
6. More than 100,000 points are calculated for each graph. You are not expected to reproduce a diagram like this on your own.

5) LOGISTIC MAPPING

1. Mapping helps show how the values generated by the logistics equation are related to each other. For example, the numbers to the right are created when "R" = 3.8
2. To Map the points, the value of X(N) is plotted on the X or horizontal axis, and X(N+1) is plotted on the Y or vertical axis.
3. The first few points are plotted for you on the graph. Keep plotting points until a pattern appears.

LOGISTICS MAPPING - X(N+1) VS X(N)

$X(N+1) = R * X(N) * (1 - X(N))$

ENTER R (.0001 TO +4) : 3.8

ENTER X(0) (.0001 TO .99) : .5

PLOT ITERATIONS STARTING WITH # 200
CHOOSE FROM 0 TO 1000

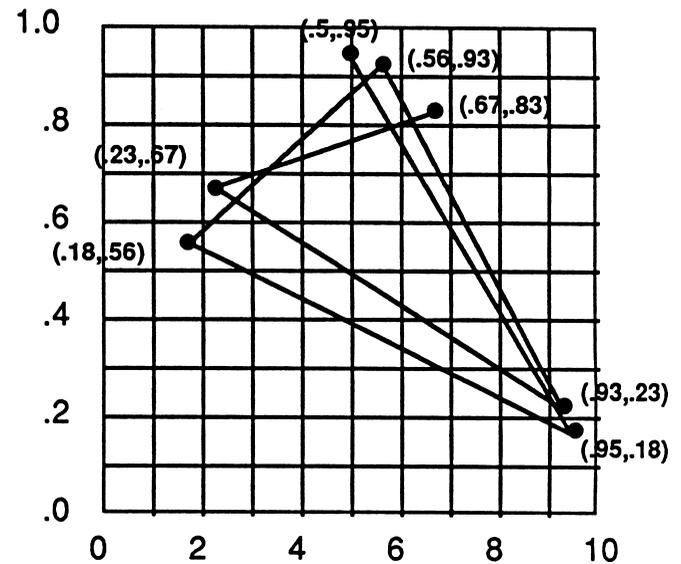
GRAPH FORM : 1) DOTS - 1000
2) LINES - 100
3) COB WEBS - 100

ENTER SELECTION : 3

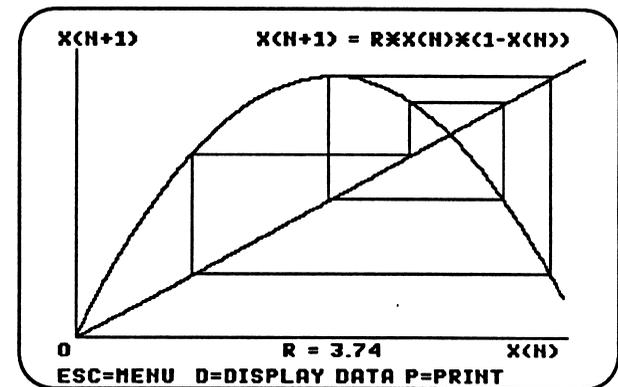
4. The Dot mapping only plots the dots on the graph. The Line mapping connects the dots in the order they are plotted. The Cob Web diagram goes from a point to the diagonal to the next point and back to the diagonal before reaching the next point.
5. Try the list of points given for the graphing section with different types of mapping. Notice that the Cob Web diagram with "R"=3.74 only shows 5 different values. The maps help distinguish chaos from periodic, repeating values.
6. 100 lines are plotted for the last two options. If only a few lines appear on the screen, the computer is just drawing the lines over and over in a periodic, repeating pattern.

$X(N+1) = R * X(N) * (1 - X(N))$ R=3.8 X0=.5

0: .5	20: .239629836
1: .95	21: .692388036
2: .1805	22: .809350006
3: .56209505	23: .58634978
4: .935347978	24: .921666119
5: .229794124	25: .274351201
6: .672557382	26: .756513954
7: .836851009	27: .699962247
8: .518819311	28: .798057379
9: .948654167	29: .612414835
10: .185095865	30: .901979038
11: .573174466	31: .335968841
12: .929652891	32: .847756359
13: .248513895	33: .490448955
14: .709668008	34: .949653354
15: .782949441	35: .181685071
16: .645770533	36: .564967303
17: .869253617	37: .933961148
18: .431876713	38: .234375343
19: .932365027	39: .681885458



Cob Web Map



4) ATTRACTORS

Attractor Menu

```

*****
*                                     *
*           ATTRACTORS               *
*                                     *
*  1) GINGERBREAD MAN                *
*  2) SIN ATTRACTOR                  *
*  3) SIN ATTRACTOR II               *
*  4) HENON ATTRACTOR                *
*  5) LORENZ ATTRACTOR               *
*  6) DESCRIPTION                    *
*  7) RETURN TO CONTENTS             *
*                                     *
*****
1, 2, 3, 4, 5, 6, OR 7

```

Gingerbread Man Parameters

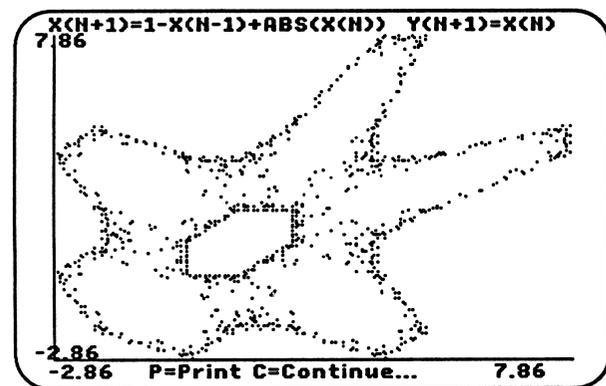
```

GINGERBREAD MAN
X(N+1)=1-X(N-1)+ABS(X(N))  Y(N+1)=X(N)

ENTER X(0) VALUE (-2 TO +2) : -.1
ENTER Y(0) VALUE (-2 TO +2) : 0

```

Gingerbread Man Graph



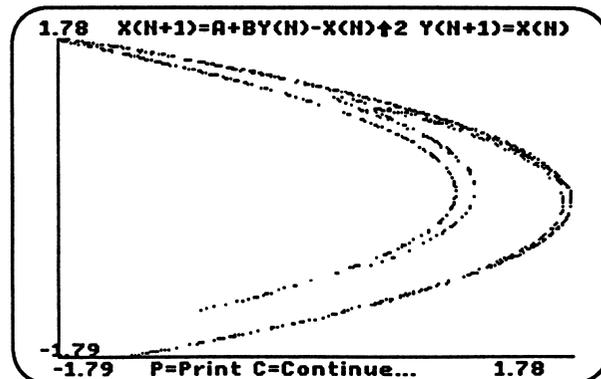
1) GINGERBREAD MAN

1. Most attractors have the same properties, so this section will concentrate on just one, the Gingerbread Man.
2. The equation for the Gingerbread Man is more complex than the logistics equation, but values are computed the same way, one step at a time.

$$X(N+1)=1-X(N-1)+ABS(X(N)) \quad Y(N+1)=X(N)$$

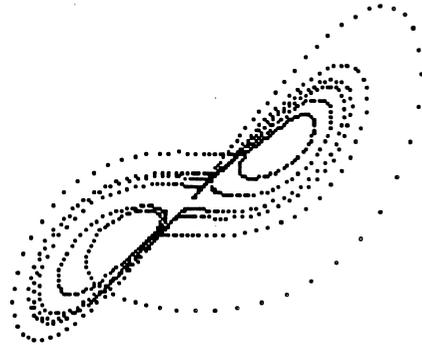
3. Set $X(0)$ to $-.1$, and $Y(0)$ to 0 to obtain the standard graph. You will need to be patient for a minute or two.
4. Other initial conditions, like $-.4, .4$, give a similar graph but with different concentrations of dots.
5. Some initial condition produce values that approach infinity, so no graph can be drawn. Other conditions produce just 6 points which are drawn over and over on the graph. Keep a record and try to find a connection in initial conditions that produce interesting patterns.
6. Select 4) HENON ATTRACTOR and enter the following parameters: $1.4, .3, 0$, and 0 . Initial conditions *close* to the pattern will be *attracted* to the curve shown below. Points further away generally produce values that approach infinity and can not be graphed.

Henon Attractor



Programming the Lorenz Attractor

1. Programming the Apple II to graph attractors is easy, but computer programming has no room for errors.
2. Start up the computer with the Data Disk in Drive 1, or any other initialized disk except your Program Disk.
3. Press down the Control key, and keep it held down while you press the RESET key. The computer will "beep" and the Applesoft prompt "]" will appear. Type NEW: HOME and press <RETURN>.
4. Type in the following. Press <RETURN> only after the last letter in a program line.
5. You may leave out any REM and all the words that come after REM.
6. If you make a typing mistake, and have not pressed <RETURN>, use the arrow keys to back up. Retype the error and all the commands after it.
7. If you discover the error later, just retype the entire line, including the line number. It pays to type slowly and accurately.



```
10 REM PLOT LORENZ ATTRACTOR
20 TEXT : HOME : REM CLEAR SCREEN
30 FR = 8 / 3 : H = .01
40 INPUT "X = "; X
50 INPUT "Y = "; Y
60 INPUT "Z = "; Z : REM INITIAL VALUES
70 HGR2 : HCOLOR = 3 : REM SET GRAPHICS
80 FOR I = 1 TO 2000 : REM SET NUMBER OF POINTS
90 XN = X + 10 * H * (Y - X)
100 YN = Y + H * (28 * X - Y - X * Z)
110 ZN = Z + H * (X * Y - Z * FR) : REM COMPUTE NEW VALUES
120 X = XN : Y = YN : Z = ZN : REM ASSIGN NEW VALUES
130 H PLOT 140 + 3 * X, 95 - 3 * Y : REM PLOT VALUES
140 IF PEEK ( - 16384 ) = 155 THEN I = 2000 : REM END IF ESC PRESSED
150 NEXT I : REM CONTINUE IN LOOP
```

1. Once you have the program typed in, type:

SAVE LORENZ

This will save the program on the disk.

2. Type RUN. Enter .1, .1 and .1 for the values of X, Y and Z.
3. A graph of the attractor will appear, one dot at a time, on the screen.
4. Try entering different initial conditions for X, Y and Z. It is possible that the computer will try to plot points off the screen. You will get an error message. If the message is not visible, type

TEXT

5. All the graphs will have the same "Butterfly" form, but the path traced by the dots will have distinctly different paths for different initial conditions.
6. If you are adventurous, you can try changing some of the numbers within the program. This can lead to problems, as well as new patterns.
7. If you change the program, and you like the change, you can save the new program with:

SAVE LORENZ 1

8. If you want to run the program later, boot up the computer and put the disk containing LORENZ in Drive 1. Type:

RUN LORENZ

9. Your program will be loaded in automatically and the RUN.

5) FRACTAL LINES

1) SNOWFLAKE - LINE

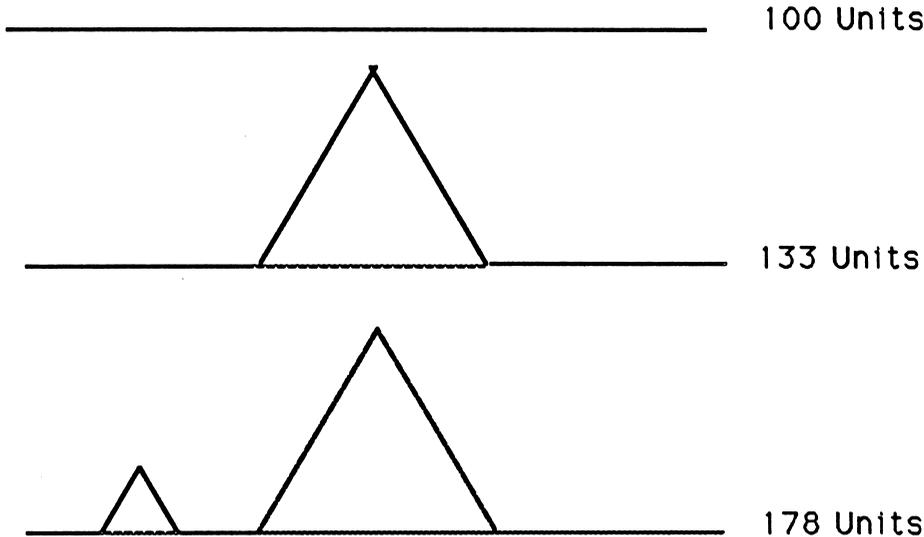
Fractal Lines Menu

1. Fractal lines are extremely rough lines constructed by adding smaller and smaller bumps to more and more sections of the line. For example, start with a single line 100 units long.
2. Divide the line into 3 even parts. Over the center part, add a triangle that has all three sides equal. Remove the center section of the original line. The total length of the new line is $(100 / 3) * 4 = 133$ units.

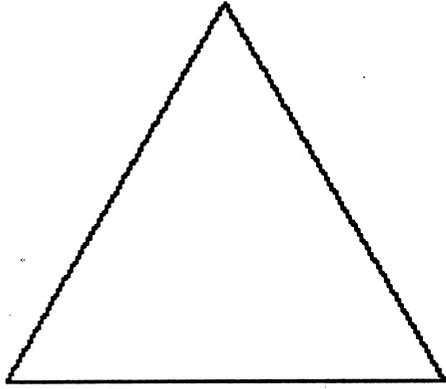
```

*****
*                                     *
*                               FRACTAL CURVES                               *
*                                     *
* 1) SNOWFLAKE - LINE               *
* 2) SNOWFLAKE - CURVE              *
* 3) SQUARE - LINE                  *
* 4) SQUARE - CURVE                 *
* 5) RECTANGLE - LINE                *
* 6) RECTANGLE - CURVE               *
* 7) PAENO - LINE                    *
* 8) PAENO - CURVE                   *
* 9) DESCRIPTION                     *
* 10) RETURN TO DISK CONTENTS        *
*                                     *
*****
    
```

1, 2, 3, 4, 5, 6, 7, 8, 9, OR 10

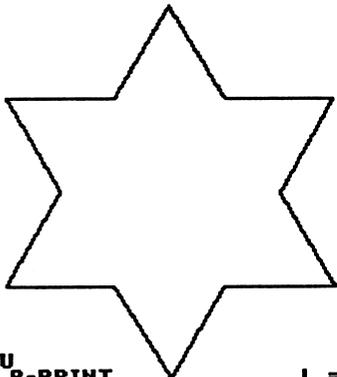
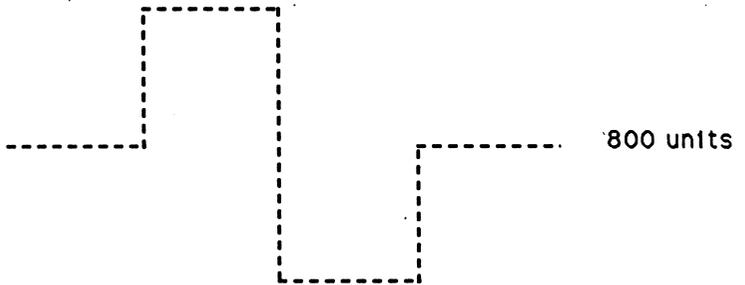


3. You must add another 3 triangles to the pattern above, bringing the total length up to 178 units.
4. The next step is to add 16 triangles, one on each line segment on the diagram above. The line will be 237 units long.
5. The lines you are drawing are becoming fractal lines—lines that are very rough and very long in a small amount of space.



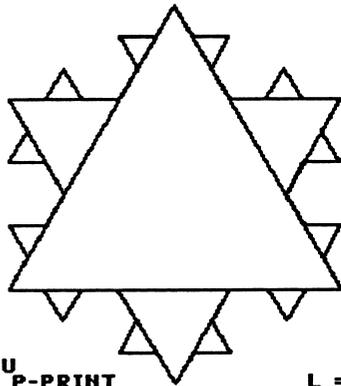
6. You can build a snowflake by repeating this procedure on a triangle. Try adding lines until you create a snowflake curve.
7. Try adding lines in the pattern shown below. On all 8 line segments, add a square on both sides of the line. Notice the length doubles every time you repeat the pattern. How do your results compare to the computer graphics?

----- 400 units



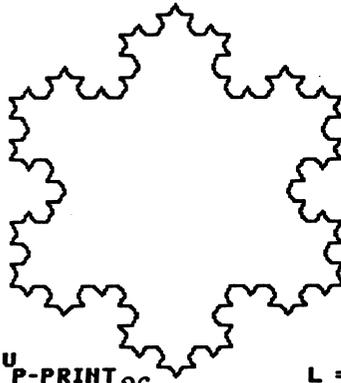
ESC-MENU
E-ERASE P-PRINT

L = 400



ESC-MENU
E-ERASE P-PRINT

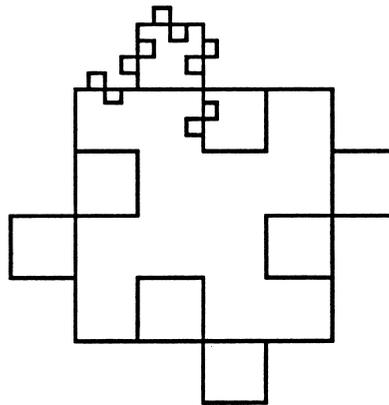
L = 533



ESC-MENU
E-ERASE P-PRINT 26

L = 711

8. Now try adding the same pattern to all four sides of a square. The pattern is started for you below. If the original sides of the square are 100 units each, how long is the line started below.



9. Try your own patterns on a piece of graph paper. How do your curves look?

6) RANDOM WALKS

Random Walk Menu

1) SIMPLE RANDOM WALK

1. The first option demonstrates the idea behind random walks. A small square bounces around the screen in a random manner. Some commands at the bottom of the screen help make the motion clearer.

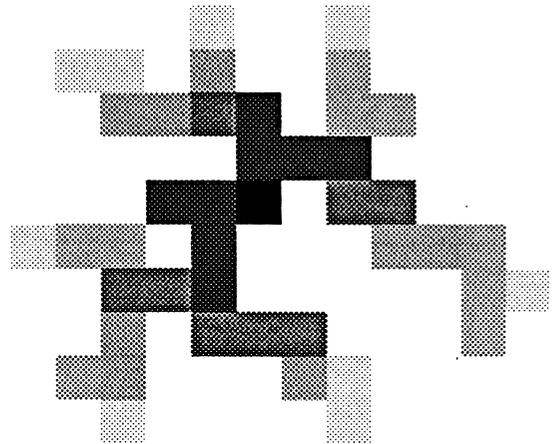
ESC-MENU T-TRAIL ARROWS-SPEED

2. As usual, pressing the <ESC> key will return you to the starting menu.
3. Pressing the letter "T" will turn the trail the dot leaves either on or off.
4. The arrow keys will slow down or speed up the motion of the square. On computers with fast speeds, the square may move way too fast for your eyes to follow.
5. Does the trail ever fill up the screen completely? How long does it take?

2) STICKY SINGLE DOT

1. Smoke particles bounce around in the air as if they were undergoing a random walk. When the smoke particles hit something, they tend to stick to it. Options 2, 3 and 4 deal with these sticky particles.
2. Particles are released and move randomly until they hit another particle. The three screens have three different starting patterns for stationary particles: A single particle, a circle, and a straight line.
3. While the particles move around the screen, you have some options at the bottom of the screen.

```
*****  
*                                     *  
*               RANDOM WALKS        *  
*                                     *  
*      1) SIMPLE RANDOM WALK        *  
*      2) STICKY SINGLE DOT         *  
*      3) STICKY LINE                *  
*      4) STICKY CIRCLE              *  
*      5) RECALL SCREEN              *  
*      6) DESCRIPTION                *  
*      7) RETURN TO DISK CONTENTS    *  
*                                     *  
*****  
1, 2, 3, 4, 5, 6, OR 7
```



ESC-END F-FREEZE

DOT: 0

Data Disk Placement

Keep the Data Disk in Drive 2
DURING FILE WORK.

PRESS <SPACE BAR> TO CONTINUE...

Recall Menu

*** DATA ON THIS DISK ***

1: CIRCLE
2: DOT
3: LINE

1) RECALL A FILE
2) DELETE A FILE
3) RETURN TO LIFE MENU

YOUR SELECTION : 1

NUMBER OF DATA FILE TO RECALL:

OR

NUMBER OF DATA FILE TO DELETE:

4. Press the <ESC> key to get back to the main menu. Press "Y" for Yes when the option appears.

END WALK (Y/N) : Y

5. Press "F" to freeze the motion of the particle. When the particles are frozen, another set of options appear at the bottom of the screen.

ESC-MENU C-CONTINUE S-SAVE DOT: 1

6. Press "C" to continue the motion with no changes.

7. Press "S" to save the screen. You will be asked to enter a name:

FILE NAME: ?

8. The file will be saved on the Data Disk.

5) RECALL SCREEN

1. Option 5 lets you recall and delete files from the Data Disk. Follow the same procedures outlined for previous menu options.

6) DESCRIPTION

1. Two pages of descriptions will explain about Random Walks.

Do It Yourself

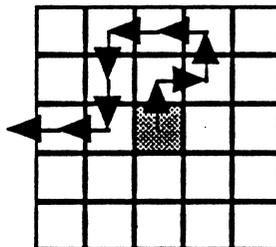
1. Random walks appear in many places in science. You don't need a computer to create random walks—you can do it yourself with a piece of graph paper and two coins.
2. You will start in the middle of the graph. Your motion will be determined by the flip of two coins.

Coin 1	Coin 2	Motion
Head	Head	Up
Head	Tail	Left
Tail	Head	Right
Tail	Tail	Down

3. The graph below shows the effect of the following coin tosses.

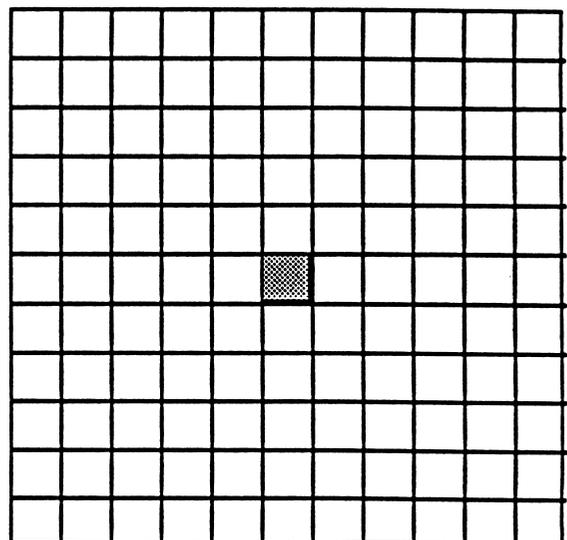
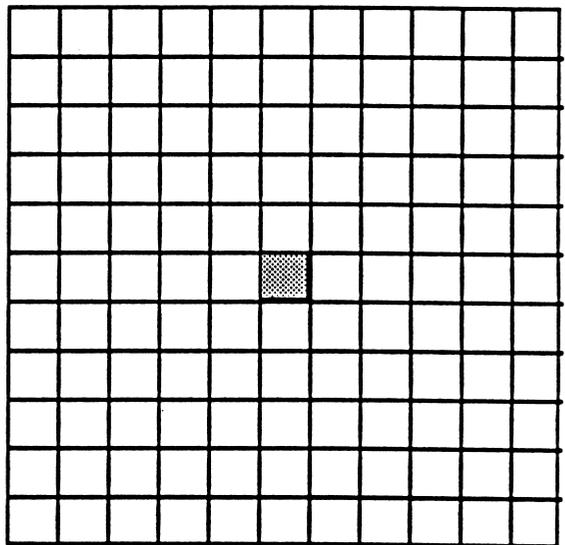
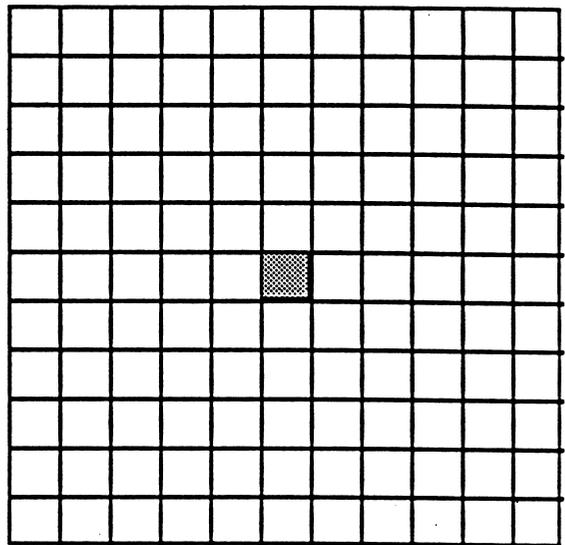
Throw Results

1	H H
2	T H
3	H H
4	H T
5	H T
6	T T
7	T T
8	H T
9	H T



After 9 throws, you have walked off the graph.

4. Try walking off the graphs to the right. How many throws does it take to get off the graph? Do the walks show any pattern? Do you ever walk in the same direction three times in a row? How often do you return to the center square before you walk off the graph? DO you ever walk off the graph?
5. Only one block is moving on the computer, but it is moving so fast that the screen shows is a 5 or 6 places on the screen. What might take you an hour to do requires only a second or two on the computer.



Recalling a Graph

```

1: PLOT MANDELBROT
2: SMALL BOMB
3: BIG BOMB

1) RECALL GRAPH
2) DELETE GRAPH
3) CONTINUE

(1, 2, OR 3) ?
3

```

1. Following the same pattern as the other programs, you can recall and delete pictures of the Mandelbrot Set.

3) RECALL MANDELBROT SET

```

1: PLOT MANDELBROT
2: SMALL BOMB
3: BIG BOMB

CHOOSE A FILE NAME
OF 15 LETTERS OR SPACES:
INFO. NAME - ?

```

3. One interesting feature of these graphs is the amount of detail available. The border between the black and white areas is the most interesting. Choosing a pure black area is not interesting, but picking coordinates of border area requires practice. Use the Low Resolution graph to test your coordinates first. If it looks interesting, try the High Resolution coordinates.
4. It takes HOURS to complete a High Resolution graph.
5. Hit the <ESC> key while the graph is being plotted, and a new menu appears. From this menu you can continue graphing, or save, print, or redisplay the graph.
6. Graphs with a great deal of black take much longer than graphs that are mostly color.

Saving a Graph

```

*****
* HI RES MENU
*
* 1) CONTINUE PLOTTING POINTS
* 2) SAVE THE GRAPH
* 3) PRINT THE GRAPH
* 4) REDISPLAY GRAPH
* 5) RETURN TO MENU
*
*****
1, 2, 3, 4, OR 5

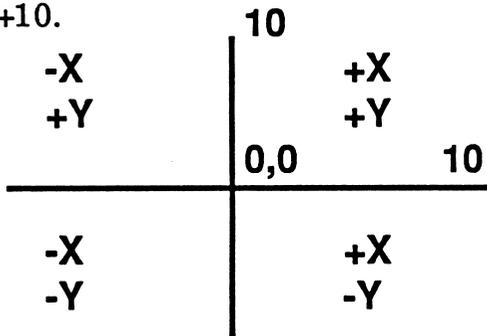
```

High Resolution Menu

8) THREE BODY ORBITS

1) CREATE & TEST ORBITS

1. Three Body Orbits lets you play with three moving objects like the sun, the earth, and the moon. You must enter the mass, the position, and the velocity of each object.
2. The mass must be between .0001 and 1000. The larger the mass, the more effect an object will have on the other bodies. Larger mass also means the object is more difficult to start moving. A mass of 500 is suitable for a sun. A planet might have a mass of 5, and a moon a mass of .5 . These numbers are only guidelines.
3. The starting or initial position of the mass is determine by the X and Y coordinates. The origin (0,0) is at the center of the screen. Values range from -10 to +10.



4. The initial velocity also requires two numbers. VX describes how fast the object is moving in the horizontal direction, and VY describes how fast it is moving in the vertical direction. Values range from -50 to +50.
5. In this example, the first object behaves like the sun. The second particle behaves like a planet. The third particle is very small and simply shoots off the screen so only the sun and a planet is left. Press the <SPACE BAR> to start the plotting. It may take a few minutes.

Three Body Orbits Menu

```

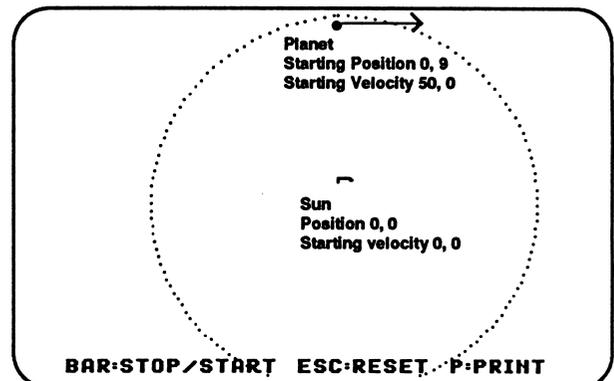
*****
*
*
*          THREE BODY ORBITS          *
*
*  1)  CREATE & TEST ORBITS          *
*  2)  DESCRIPTION                    *
*  3)  RETURN TO DISK CONTENTS       *
*
*****
1, 2, OR 3
    
```

Initial Conditions

INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	500	0	0	0	0
2	5	0	9	50	0
3	.0001	10	10	50	50

.0001<MASS<1000 -10<X<+10 -50<VX<50
 <SPACE BAR>:PLOT <P>:PRINT <R>:REENTER

Motion of Objects

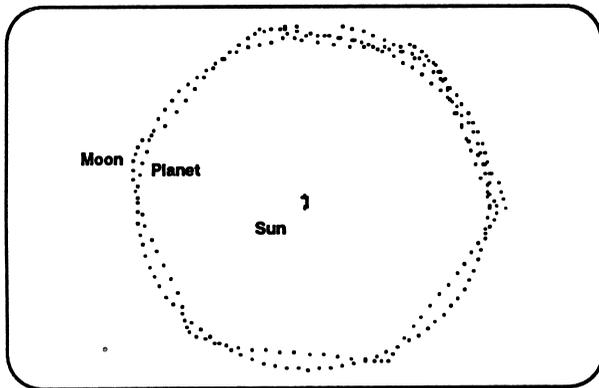


Sun, Planet, and Moon

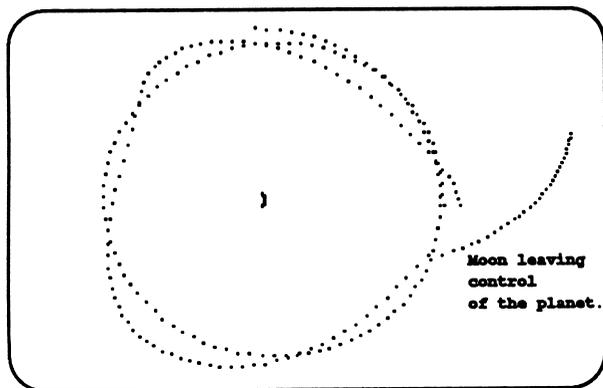
INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	500	0	0	0	0
2	5	8	0	0	50
3	.5	8.7	0	0	35

.0001<MASS<1000 -10<X<+10 -50<VX<50
 <SPACE BAR>:PLOT <P>:PRINT <R>:REENTER

Sun, Planet and Moon



Unstable Moon Orbit



- In our solar system, it is common for planets with moons to be in orbit around the sun, and to stay in orbit for billions and billions of years. It is NOT so easy to set up this system on the computer.
- The sun remains in the center of the screen. The planet moves in a fairly circular orbit around the sun. The moon weaves in and out of the planet's orbit. The moon is really in orbit around the planet, but its path through the solar system is like a wave about the planet's orbit.
- Slight variations in the orbit of the moon might be caused by collisions with comets. How would a small change in position or velocity change the motion of the moon? Try entering slightly different values for the moon to see what happens.

9.

INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	500	0	0	0	0
2	5	8	0	0	50
3	.5	8.7	0	0	30

.0001<MASS<1000 -10<X<+10 -50<VX<50
 <SPACE BAR>:PLOT <P>:PRINT <R>:REENTER

Changing only the velocity of the moon completely changes the long term motion of the moon.

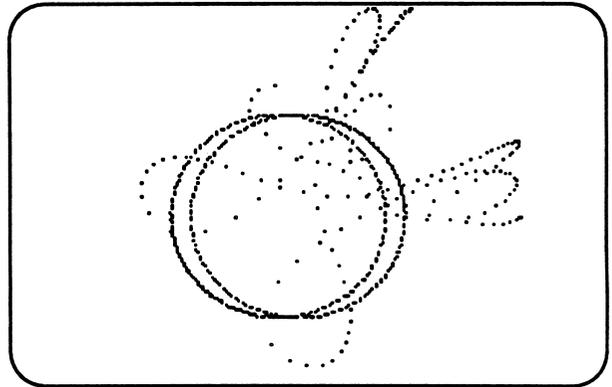
Two Sun Solar System

10. Some solar systems have two suns. The data to the right sets up two suns orbiting around each other. The third object is a small asteroid coming in from outer space.
11. Again, try making small changes in the initial conditions of the incoming particle. If a small change makes an unpredictable change in the motion of the object, the motion is Chaotic.

INPUT TABLE FOR SYSTEM 1					
OBJECT	MASS	X	Y	VX	VY
1	500	5	0	0	30
2	500	-5	0	0	-30
3	.5	10	0	-1	-10

.0001<MASS<1000 -10<K<+10 -50<VX<50
<SPACE BAR>:PLOT <P>:PRINT <R>:REENTER

Chaotic Motion



Rule

Try It Yourself
on these pages.

Sum

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

New Value

--	--	--	--	--	--	--	--	--	--

1

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

2

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

3

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

4

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

5

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

6

--	--	--	--	--	--	--	--	--	--

Rule

Sum

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

New Value

--	--	--	--	--	--	--	--	--	--

1

--	--	--	--	--	--	--	--	--	--

Sum

--	--	--	--	--	--	--	--	--	--

2

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Sum

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3

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Sum

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4

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Sum

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5

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Sum

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6

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Rule

Sum

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

New Value

--	--	--	--	--	--	--	--	--	--

1

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Sum

○	○	○	○	○	○	○	○	○	○
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2

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Sum

○	○	○	○	○	○	○	○	○	○
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3

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Sum

○	○	○	○	○	○	○	○	○	○
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4

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Sum

○	○	○	○	○	○	○	○	○	○
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5

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Sum

○	○	○	○	○	○	○	○	○	○
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6

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Rule

Sum

0	1	2	3	4	5	6	7	8	9
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New Value

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1

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Sum

○	○	○	○	○	○	○	○	○	○
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2

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Sum

○	○	○	○	○	○	○	○	○	○
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3

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Sum

○	○	○	○	○	○	○	○	○	○
---	---	---	---	---	---	---	---	---	---

4

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Sum

○	○	○	○	○	○	○	○	○	○
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5

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Sum

○	○	○	○	○	○	○	○	○	○
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6

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